1602/55.

ANALYSIS FLUXIONUM.

AUCTORE GUIL. HALES, D. D.

RECTORE DE KILLESANDRA,

ET NUPER TRIN. COLL. DUBLIN SOCIO, AC LINGUARUM ORIENTALIUM PROFESSORE.

Sanctos auſus recludere fontes. VIRG.

LONDINI:
TYPIS J. DAVIS, IN VICO DICTO CHANCERY-LANE;
PROSTAT APUD JOHANNEM WHITE, BIBLIOPOLAM, IN VICO DICTO FLEET-STREET.
MDCCC.

BRITISH MUSEUM
29 FEB 68

INDEX CAPITUM.

PARS PRIMA.

§ 1. *De Fluxionum Inventione* - - - - - - - Pag. 1
§ 14. *De Objectionibus hactenùs Impugnantium Evidentiam et Certitudinem Methodi Fluxionum* - - - - - - - - - - - 6
§ 28. *De Methodo Rationum Primarum et Ultimarum, ſeu Limitum* - - 15
§ 39. *Colſon's Account of the Method of Fluxions* - - - - - 19
§ 43. *Maclaurin's Account of the Method of Fluxions* - - - - 21
§ 47. *De Applicatione Methodi Rationum Primarum et Ultimarum, ad Quadraturam Curvarum, atque ad Mechanicam Rationalem* - - 23
§ 64. *De Fluxionum Elementis Primis* - - - - - - - 28

PARS SECUNDA.

§ 78. *Definitiones* - - - - - - - - - - - - Pag. 35
§ 79. *Axiomata* - - - - - - - - - - - - - 36
§ 80. *De Notatione Fluxionali* - - - - - - - - 37
§ 83. *De Methodo Fluxionum Directâ* - - - - - - - 38
§ 102. *De Fluxionibus Superiorum Ordinum* - - - - - - 47
§ 106. *De Theoremate Binomio* - - - - - - - - 49
§ 112. *De Methodo Fluxionum Inverſâ* - - - - - - - 52
§ 118. *De Correctione Fluentium* - - - - - - - - 53
§ 119. *De Fluxionibus diverſas Fluentes admittentibus* - - - - - 54

§ 120. *De Fluxionibus Logarithmorum* - - - - - - Pag. 55
§ 128. *De Fluxionibus Quantitatum Exponentialium* - - - - - 57
§ 129. *De Logarithmis Imaginariis* - - - - - - - - 58
§ 132. *Formulæ Fluxionum atque Fluentium* - - - - - - 59
§ 133. *De Formularum Applicatione* - - - - - - - 61

APPENDIX PRIMA.

§ 135. *De Analysi Antiquâ* - - - - - - - - Pag. 65

APPENDIX SECUNDA.

§ 142. *De Æthere Vibratorio* - - - - - - - - Pag. 73
§ 151. *De Sensorio, et de Vi Sentiendi* - - - - - - 79
§ 155. *Maclaurin's Last Words* - - - - - - - - 82
§ 157. *D'Alembert's Mournful Reflexions* - - - - - - 85
§ 162. *De Philosophiâ Atomicâ Priscorum Chaldæorum et Hebræorum* - 88
§ 174. *De Æthere Antiquo, Deo Ficto Gentilium* - - - - - 93
§ 196. De Ente Supremo - - - - - - - - - 102
De Nominibus Dei Primitivis, Not. (*m*) - - - ib.

BRITISH MUSEUM 29 FEB 68

ANA-

FRANCISCO MASERES,

CURIÆ SCACCARII REGIS MAGNÆ BRITANNIÆ IN ANGLIÂ BARONI QUINTO;

ANALYSTÆ EGREGIO,

MATHESEOS PATRONO,

VIRO BONO,

AMICO FIDO;

VINDICIAS HÁSCE NEWTONIANAS—

METAPHYSICAS ET MATHEMATICAS,

PHYSICAS ET THEOLOGICAS;—

QUÂ PAR EST OBSERVANTIÂ,

GRATUS LUBENSQUE

D. D. D.

GUILIELMUS HALES.

BRITISH MUSEUM
29 FEB 68

PRÆFATIO.

INTER THEORIAM ÆQUATIONUM ALGEBRAÏCARUM *atque* FLUXIONALIUM *tanta intercedit Analogia, eædèmque tam arctâ affinitate, imò cognatione, inter ſeſe conſociantur et conjunguntur; ut abſque hujus ope, illa jure imperfecta, et quaſi manca, exiſtimari debeat: Nam per calculum* FLUXIONALEM, *ſeu, ut ab exteris deſignatur,* DIFFERENTIALEM, *(vix niſi nomine et notatione à ſe invicèm diſcrepantes) optimè promovenda eſt* ALGEBRA. *Sicut unico patere poterit exemplo,* THEOREMATIS BINOMII, *quod, licet indoles ejus ſit omninò fluxionalis, omnès tamen* MATHESEOS *regiones permeat et adauget. Hanc igitùr* ANALYSIN FLUXIONUM, ANALYSI ÆQUATIONUM *pridem editæ* (*), *planè gemellam, edere jam libet; ut pro virili fungar munere interpretis fidi* ANALYSEOS ANTIQUÆ *illius, veræ et genuinæ*:—" Quæ cum *Algebrâ Specioſâ* facilitate contendit, evidentiâ verò et demonſtrationum elegantiâ eam longè ſuperare videtur;"—*Teſte, peritiſſimo Geometrâ ſimùl ac Analyſtâ,* EDMUNDO HALLEY; *qualem eam optimè exponit* PAPPUS ALEXANDRINUS *(in* APPENDICE, I. § 136.) *ex ſcriptis Veterum Analyſtarum*:—ARCHIMEDIS, in Quadraturâ *Parabolæ*; APOLLONII, in tractatu de Sectione *Rationis*; EUCLIDIS, in Libris *Datorum* et *Poriſmatum*; ERATOSTHENIS, de Mediis *Proportionalibus*, &c.—*Hanc verò* ANALYSIN ANTIQUAM *pænè amiſſam*, (CARTESIO, *ejúſque ſectatoribus, haud minùs* MATHESIN *quàm* PHILOSOPHIAM *inquinantibus, adhibendo demonſtrationes, antiquis quidem breviores*;

(*) *Tractatum priorem* (ANALYSIN ÆQUATIONUM) *impenſis propriis edidi anno* 1784; *hunc verò, pro ſingulari ſuo in* MATHESIN, PHILOSOPHIAMQUE BRITANNICAM *ſtudio, impenſis ſuis editurus eſt vir cl.* FR. MASERES, *vel ipſo flagitante. Neque, ut ſpero, in amici digniſſimi modeſtiam peccâſſe cenſebor, ſi in lucem, (etiam illo invito), excerpta quædam deprompſerim ex epiſtolâ ejus, hanc rem ſpectantia; Sept.* 17. *anno* 1799 *datâ*:

—" I am glad to find that you have been ſo actively and uſefully employed, ſince your return to IRELAND, as in reviſing and enlarging your *Analyſis Fluxionum*; which I hope you will ſoon publiſh. If you will ſend it to me, I will print it for you, either in the *Fifth* Volume of the *Scriptores Logarithmici*, or by itſelf;—or half the number of copies in the *Scriptores Logarithmici*, and the other half by themſelves, as you ſhall direct. And I believe that laſt manner of printing it would be the beſt:—that is, to print 250 copies in the *Scriptores Logarithmici*, and another 250 ſeparately; as, by this way, it will come *ſooner* into the hands of the Learned."——

—" I wiſh you every opportunity of propagating your taſte for ſound literature and ſcience; and thereby of becoming *eminently uſeful* to your COUNTRY: than which, *nothing can afford* you *greater ſatisfaction*."

See § 4, Note (*a*); § 17, Note (*d*); and § 131.

at longè laxiores atque infirmiores) curioſâ felicitate reſtituit, et inſtauravit et perfecit NEWTONUS, *Veterum evidentiam et certitudinem, Recentiorum brevitatem et facilitatem, rariſſimâ conjunctione aſſecutus; ſi modò rite expendatur et probè intelligatur imperatoria brevitas quâ* METHODUM FLUXIONUM *delibando tranſcurrit Inventor ille ſagaciſſimus.*

Hujus Methodi fons et origo eſt Doctrina RATIONUM PRIMARUM ET ULTIMARUM; *cujus ope demonſtrantur* LEMMATA *illa præclara ſeu* PRINCIPIA MATHEMATICA, *perbrevia et ſubtiliſſima, in primâ ſectione doctiſſimi et celeberrimi illius libri cui titulus,* Philoſophiæ Naturalis Principia Mathematica. *Hæc verò Doctrina, qualem ipſe auctor eam orbi litterato in diverſis ſuis ſcriptis propoſuit, pulcherrimam exhibet ſynopſin paritèr et extenſionem* METHODI EXHAUSTIONUM *à veteribus geometris uſurpatæ et excultæ, quam vir mirandâ ſagacitate præditus uno eodémque tempore in compendium redegit et ad novas et maximè arduas ſpeculationes feliciſſimè promovit et applicuit.*

Cæterùm fatendum eſt hanc Doctrinam RATIONUM PRIMARUM ET ULTIMARUM, *ex intimis* METAPHYSICÆ *fontibus hauſtam, et* "quâ potuit brevitate" *à* NEWTONO *traditam, obſcuritate non facilè diſcutiendâ involvi: quocircà expoſitio ejuſdem, ſatìs lucida, evidens, et accurata (non obſtantibus tot tantíſque Interpretibus Analyſeos Newtonianæ) hactenùs deſiderari videtur; éſtque conditio ſine quâ non clarè cernere atque animo complecti licebit fundamentum ſolidum et nullis cavillationibus dimovendum, cui adſtruitur tota* THEORIA FLUXIONUM, *neque diluere objectiones doctorum quorundam virorum, qui etiàm his ſeris temporibus ejuſdam evidentiam et certitudinem impugnant, ſive ex inſcitiâ vel imperitiâ, vel denique ex invidiâ et partium quodam ſtudio, oriundas.*

Itaque, ne falſis criminibus ulteriùs premi videatur Methodus Fluxionum, in PRIMA PARTE, *auſus ſum fontes ipſos* METAPHYSICOS *recludere ex quibus derivantur* PRINCIPIA MATHEMATICA FLUXIONUM, *in* SECUNDA PARTE; *Analyſeos Antiquæ et veræ quaſi cuſtos rigidúſque Satelles; et magni mirandíque* ANALYSTÆ BRITANNICI *æquus et candidus vindex; excuſſis quàm fideliſſimè et quàm accuratiſſimè potuerim, objectionibus Analyſtarum magni nominis,* BERKELEY, LANDEN, D'ALEMBERT, TORELLI, DE LA GRANGE, &c. et nuperrimè, CENSORIS (ut ferunt, *Profeſſorii* et *Academici,*) apud THE MONTHLY REVIEW, *Appendix, April* 1798, *Vol.* 28. N. S.

In hâc Diſquiſitione, ſubtiliſſimâ ſane et reconditiſſimâ, et inter laborem "tardè eruendi quæ tam altè jacent," *Demonſtrationem Theorematis Binomii, à doctiſſimo viro, Johanne Landen, inventam, (quam ego dudùm admiratus fueram et in* ANALYSI ÆQUATIONUM *adhibueram et excolueram,)*

eram,) iterùm ad examen revocare viſum eſt ; et, cum iſta demonſtratio vix, et ne vix quidem, in principiis in quibus fundatur, à computo fluxionali diſcrepare videtur, aliam demonſtrationem ejuſdem Theorematis omninò, novam, et in principiis Methodi Fluxionum fundatam, huic Landenianæ ſubſtituendam eſſe judicavi ; quæ quidem eſt ipſâ longè facilior, et ſimplicior, et ad naturam Theorematis demonſtrandi magìs accommodata, utpote quæ derivatur ex analogiâ quâdam elegantiſſimâ, quæ intercedere obſervatur inter SERIEM BINOMIAM *et* SERIEM FLUXIONUM ORDINUM SUPERIORUM, *quarum ſerierum termini correlativi ſunt ad ſe invicèm ſemper in datâ quâdam ratione.*

His VINDICIIS NEWTONIANIS—METAPHYSICIS *et* MATHEMATICIS—in APPENDICE II. *alias quoque*—PHYSICAS *et* THEOLOGICAS—*ſubjungere viſum eſt ; ut laus debita illi* PHILOSOPHO CHRISTIANO, *integra ſervetur et illibata ; quam inquinare non verentur, falsò atque ignavè,* MINUTI *quidam* PHILOSOPHI, *hoc* " SÆCULO *perperàm ſanè ab iis, eorùmque ſectatoribus,* RATIONIS *dicto. Pudètque inter hujuſmodi obtrectatores, inveniri ipſius* NEWTONIANÆ PHILOSOPHIÆ (ad THEOLOGIAM *ſobriè et humilitèr hactenùs ancillantis,) non ſolùm Diſcipulum ſed etiàm Profeſſorem,* ROBISON, *apud* EDINBURGENSES ; *temerè inculpantem theoriam illam ſublimem at longè ſubtiliſſimam* " De pulſibus *medii ætherii*, vibrando propagatis à *corporibus tremulis* ad *ſenſorium* uſque ; ibique, à *ſubſtantiâ ſenſitivâ*, ſeu *vi ſentiendi*, *intelligendi*, et *agendi* perceptis :"—*cujus objectiones penitiùs et fuſiùs, ex* NEWTONI *ſcriptis, et teſtimoniis* PHILOSOPHIÆ PRIMARIÆ *et* SACRÆ, *antiquitùs in honore habitæ, in hâc ſecundâ Appendice excutiuntur.*

Quàm cautè verò, et quàm timidè procedendum in luce ſubluſtri Analyſeos ſublimioris,—NATURAM DIVINAM HUMANAMQUE *explorantis, ideáſque claras et determinatas* INFINITATIS *arripere geſtientis,—monet* MACLAURIN *optimus :*

" In reaſoning concerning *finite* quantities, WE apprehend that GEOMETRICIANS cannot be too ſcrupulous in admitting or treating of *Infinites* ; of which our ideas are *ſo very imperfect.*—PHILOSOPHY probably will always have its *myſteries* ; but theſe are to be avoided in GEOMETRY (*),

and

(*) GEOMETRY has its *myſteries*, no leſs than PHILOSOPHY and RELIGION : Witneſs the elementary doctrine of *Parallels*, reſting on the diſputed foundation of the *eleventh* axiom of *Euclid*, the intuitive evidence of which is denied by *Tacquet* and others ; and the *quadrature of the circle*, which can only be ſolved by approximation, ſtill remains the reproach of Geometricians. In ALGEBRA, the *irreducible caſe* of cubick equations exhibits two of the roots under an impoſſible form, when all are real : which, notwithſtanding, can be accurately extracted in ſome particular caſes ; and in the reſt approximately, by means of the *Binomial Theorem.* And in FLUXIONS, too, the famous *Problem of the Three Bodies*, ſo neceſſary to determine the accelerations or retardations of the motions of the planets and the derangements

and we ought to guard againſt abating from its ſtrictneſs and evidence the rather, that *an abſurd* **PHILOSOPHY** *is the natural product of a vitiated* **GEOMETRY.**"

KILLESANDRA,
Oct. 23, 1799.

---

ments of their orbits, by their mutual attractions, (of which approximate ſolutions have been given by *Euler*, *Clairaut* and *D'Alembert*,) is thus repreſented by the firſt of theſe three great men, with all the candour of a profound Mathematician:—"*Hujus problematis enodatio completa omnes* ANALYSEOS *vires tranſcendere videtur.*"—The difficulty conſiſts in integrating three differential equations of the ſecond order; or, in the language of Newton's doctrine of Fluxions, in finding the fluents of three fluxionary equations of the ſecond order.—Indeed the whole theory of the *Reſolution of the Higher Equations* in *Algebra*, and the *Inverſe Method of Fluxions*, which correſponds thereto, is extremely imperfect ſtill, in this OUR INFANCY OF SCIENCE.—See *Maclaurin's* dying words, *Appendix*, II. § 162.

BRITISH MUSEUM 29 FEB

ANALYSIS FLUXIONUM.

PARS PRIMA.

DE FLUXIONUM INVENTIONE.

§ 1. *THEORIÆ Fluxionum* præſtantia inſignis, omnis *Matheſeos* fines peragrantis, promoventis et illuſtrantis, prorsùs poſtulare videtur ut præmittantur aliqua, more hiſtorico, de ejus inventione; qualia extant in *Commercio Epiſtolico Johannis Collins* et aliorum, anno Domini 1712, edito, atque iterùm auctiùs anno Domini 1722; in *Raphſon*'s *Hiſtory of Fluxions*; in diverſis editionibus *Principiorum* Newtoni; *Hutton*'s *Mathematical Dictionary*.; et *Monthly Review*, *Append.* Vol. 28, 1799.

2. Utrum *Newtono* an *Leibnitzio* tribuenda ſit laus hujus inventionis, et uter horum eam priùs meditando extuderat; diù atque acritèr conteſtatum eſt inter fautores *calculi fluxionalis* et *calculi differentialis*, vix ab invicèm niſi merâ notatione diſcrepantium; et *adhuc ſub judice lis eſt*.

3. Veruntamen hanc controverſiam penitiùs inſpicienti, et ſine partium ſtudio perpendenti, ſatìs conſtabit, utrumque horum illuſtrium Geometrarum pro ejuſdem inventore jure cenſeri poſſe; utrumque, præcellenti ingenio et mirâ ſagacitate præditum, utrumque eodem "*labore improbo*," et "*cogitandi vi indefeſſâ*" functum, ex iiſdem fontibus ſcientiæ utrique patentibus, tandem eadem *elementa metaphyſica*, eadem *principia mathematica*, methodi ſive *fluxionalis* ſive *differentialis*, hauſiſſe; atque ſectatoribus ſuis delibâſſe, principio, carptìm, ac ſine demonſtrationibus edita. Háſce quidem *Newtonus* poſteà, obitèr tantùm, et quaſi invitus, longo poſt tempore ſupplevit, parcè et "*quâ potuit brevitate*," in *Lemmate Secundo Lib. II. Princip.*; in *Quadraturâ Curvarum*; et in *Analyſi per Æquationes numero terminorum infinitas*:—Hanc curam *Leibnitzius* (vir omnimodæ ſcientiæ avidus, et à Geometriæ ſtudio nimìs ſæpe ad alias ſpeculationes ingenii vires transferens) prorsùs neglexit, diſcipulíſque ſuis celeberrimis, *Jacobo* et *Johanni Bernoulli*, eorúmque ſucceſſoribus *Euler*, *D'Alembert*, *La Grange*, et *La Place*, legavit; qui munus ſibi demandatum, demonſtrandi et in immenſum promovendi fines fluxionum, longè ultrà limites *Newtonianos* et *Leibnitzianos*, ingenti cum emolumento ipſius Matheseos, exſecuti ſunt.

4. Gloriabatur certè *Leibnitz*,—"ſe *Newtono* nil debuiſſe;" nec inficiabatur *Newtonus:* pro ſuâ modeſtiâ inexpugnabili nihil ſibi arrogans, et tantùm poſtulans *jus æquum partitionis*, patiens et commodus; "cui omnium mortalium minimè neceſſarium fuit "*laudem aliunde mutuari:—Hanc ſibi ipſi peperit.*" (*a*). Audiamus ipſum, in *primâ editione* PRINCIPIORUM, anno 1687, p. 253. de *Fluxionum inventione* differentem:

"*In literis quæ* MIHI *cum Geometrâ peritiſſimo* GULIELMO, GOTHOFREDO, LEIBNITIO *annis abhinc decèm* [1677] *intercedebant, cum ſignificarem me compotem eſſe methodi determinandi Maximas et Minimas, ducendi Tangentes, et ſimilia peragendi; quæ in terminis ſurdis æquè ac in rationalibus procederet; et literis tranſpoſitis hanc ſententiam continentibus*—"Datâ æquatione, quotcunque fluentes quantitates involvente, fluxiones invenire"—*eandem celarem: reſcripſit vir clariſſimus* 'ſe quoque in ejuſmodi methodum incidiſſe;' *et methodum ſuam communicavit, à meâ vix abludentem, præterquam in verborum et notarum formulis,*" ["et *ideâ generationis quantitatum*"—inſeruntur hæc quatuor verba in editione ſecundâ PRINCIPIORUM anno 1713]—"*Utriuſque fundamentum continetur in hoc lemmate.*" [II. *Lib*. II. PRINCIP.]

5. Et certè fatendum eſt, ſemina non obſcura *methodi ſuæ differentialis*, poſuiſſe *Leibnitzium*, in *Theoriâ Notionum Abſtractarum*, Academiæ Regiæ *Pariſienſi* dicatâ, anno 1671; antequam cum *Newtono* commercium aliquod epiſtolicum inſtituiſſet. Hanc methodum poſteà publici juris fecit, in epiſtolâ quâdam in *Actis Lipſ*. Jan. 21, 1677, editâ, verùm abſque demonſtratione prolatam; quam poſteà ſuppleverunt, celeberrimi fratres, *Daniel et Johannes Bernoulli*, famæ *Leibnitzianæ* vindices acerrimi, iiſdémque fontibus analyſeos ſublimioris haud levitèr imbuti.

(*a*) Dr. Halley, in his letter to Mr. *Newton*, dated May 22, 1686, informing him that the *Royal Society* had reſolved to print his "*Incomparable Treatiſe* intitled PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA," at their own charge, in a large quarto of a fair letter—"judging that ſo excellent a work ought not to have its publication *delayed*"—concludes with ſtating Mr. *Hooke*'s claim—"as having ſome pretenſions upon the invention of *the rule of gravity being reciprocally as the ſquares of the diſtances from the centre.* He ſays you had the notion from him, though he owns the demonſtration of the *curves* generated thereby to be wholly your own.—How much of this is ſo, you know beſt; as likewiſe, what you have to do in this matter. Only Mr. *Hooke* ſeems to expect you ſhould make ſome mention of him in the Preface, which it is poſſible you may ſee reaſon to prefix."

"I muſt beg your pardon that it is I that ſend you this ungrateful account; but I thought it my duty to let you know it, that ſo you might act accordingly: *being in myſelf fully ſatisfied, that the greateſt candour imaginable is to be expected from a perſon who has of all men the leaſt need to borrow reputation.*"

See the whole letter, and the proceedings of the Royal Society on this buſineſs, in *Birch's Hiſtory of the Royal Society*, Vol. IV. p. 484.

In conſequence of this unfounded claim of Dr. *Hooke*, and his own averſion to controverſy, *Newton* withdrew the third book of the *Principia*, which treated of the *Syſtem of the World* in a popular way; wiſhing not to let it go abroad into the world, without a ſtrict and rigorous demonſtration; as it appeared afterwards:—

"*De hoc argumento compoſueram librum tertium* [DE MUNDI SYSTEMATE] *methodo populari, ut à pluribus legeretur.* Sed quibus *principia* poſita *ſatis intellecta* non fuerint, ii *vim conſequentiarum* minimè percipient, neque *præjudicia* deponent, quibus à multis retrò annis inſueverunt: Et propterea, ne res in *diſputationes* trahatur, ſummam libri illius tranſtuli in Propoſitiones, more mathematico, ut ab iis ſolis legantur qui *principia* priùs evolverint." *Præfat.* in *Lib.* III. *Princip.*

BRITISH MUSEUM 29 FEB 68

6. Nec extant teſtimonia, *Newtonum* methodum ſuam fluxiónum cuivis communicâſſe ante annum 1672; regerit enim Newtonus, in brevi illâ hiſtoriâ de fluxionum inventione, *Schol. Lem.* II. *Lib.* II. *Princip.* 3tiæ edit. 1726, p. 246, in his verbis.

"In epiſtolâ quâdam ad *D. Collinſium* noſtratem 10 Decemb. 1672 datâ, cum deſcripſiſſem *methodum Tangentium* quam ſuſpicabar eandem eſſe cum methodo *Sluſii* (tum nondum communicatâ), ſubjunxi:"

—'*Hoc eſt unum particulare, vel* COROLLARIUM *potiùs*, METHODI GENERALIS *quæ extendit ſe citrà moleſtum ullum calculum, non modò* AD DUCENDAS TANGENTES *ad quaſvis curvas, ſive geometricas ſive mechanicas; vel quomodocunque rectas lineas, aliaſve curvas, reſpicientes; verùm etiàm ad reſolvendum alia abſtruſiora problematum genera* DE CURVITATIBUS, AREIS, LONGITUDINIBUS, CENTRIS GRAVITATIS CURVARUM, &c. *Neque (quemadmodum* HUDDENII *methodus de* MAXIMIS *et* MINIMIS) *ad ſolas reſtringitur æquationes quæ quantitatibus ſurdis ſunt immunes.* HANC METHODUM *intertexui alteri iſti* [DE ANALYSI PER ÆQUATIONES NUMERO TERMINORUM INFINITAS, *quam anno* 1671 *de his rebus ſcripſeram*] *quâ æquationum exegeſin inſtituo reducendo eas ad ſeries infinitas.*"—

7. Alteram quoque epiſtolam, 13 Jun. 1676, ſcripſit *Newtonus* ad *Oldenburgum*, cum *Leibnitzio* communicandam, in quâ celeberrimum *Theorema Binomium*, primò, abſque demonſtratione propoſuit, (nec unquam poſteà demonſtratione ipſe munivit) cujus ope nimirùm, "*æquationum illam exegeſin*" potiſſimùm, inſtituerat. Hoc autem, ex intimis penetralibus fluxionum hauſtum, *Newtono*, ſaltèm ante *Peſtem Londinenſem*, anno 1665, innotuiſſe, anteà memoravimus, ANALYS. ÆQUAT. p. 33. not. Vide quoque § 96 hujus.—Cæterùm hæ duæ epiſtolæ, *Leibnitzianam* illam Jan. 21, 1677, tempore antegreſſæ, *primi inventoris* honorem *Newtono* ſervare videntur; nec inficiabitur æqua poſteritas.

8. De famâ *Leibnitzii* plus æquo forsàn detrectans, cœtus Analyſtarum *Anglicorum* adjudicavit, "*That in the letter of 10th December* 1672, from *Newton* to *Collins*, the method of fluxions was *ſufficiently* deſcribed to any *intelligent* perſon."—Verùm, ſi "hæc methodus *ſatis* in iſtâ epiſtolâ *deſcripta fuit ut ab aliquo harum rerum modicè perito poſſet intelligi*, quæri poteſt," cur *Newtonus*, anno 1677, ad æmulum ſuum "*intelligentiſſimum* certè atque *peritiſſimum*" *Leibnitzium* ſcribens, anagrammate quodam *methodum ſuam fluxionum directam* diutiùs celaret?—Diſputationes ſanè quæ adhuc, etiàm noſtro ævo, vigent de evidentiâ et certitudine hujus methodi, inter Analyſtas perſpicaciſſimos, ſcholæ tam *Newtonianæ* quam *Leibnitzianæ*, (non obſtantibus, vindiciis, expoſitionibus, et commentariis diſcipulorum *Newtoni* celeberrimorum) ſatìs oſtendunt cur *tam tardè eruantur, quæ tam altè jacent.*—*Seneca.*

Adde quod, ſi verum fuiſſet, quod ex ſcriptis *Newtonianis* primos theoriæ ſuæ igniculos elicuerat *Leibnitzius*, haud mediocris laus idcircò illi concedenda videretur, qui arcanum moleſtiſſimum et longè difficillimum in lucem ex tenebris proferre poterat:

Quem vituperare ne inimici quidem poſſunt, niſi ut ſimùl laudent.

9. Veruntamen tutò colligere licet, neque *Newtonum* neque *Leibnitzium* pro inventore abſoluto hujúſce methodi habendum eſſe; nam ſi manifeſtò conſtat,

utrumque methodum ſuam à methodo *ducendi Tangentes Barrovianâ* mutuâſſe; nonne utrique laudem *pleni perfectique ab integro inventoris*, detrahet æquus controverſiæ judex?

Audiamus *Leibnitzium* in epiſtolâ quâdam April 9, 1716, datâ:

“ M. *Newton* hazarde icy un accuſation qui va tomber ſur lui-même: Il prétend que ce que j'ay écrit pour lui à M. *Oldenbourg* en 1677 *eſt un déguiſement de la méthode de M. Barrow*. Mais, comme M. *Newton* avoue dans la page 253 et 254 de la première édition de ſes principes:”—‘ *Me*, [*Leibnitzium*] *ipſi* [*Newtono*] *tunc methodum communicâſſe à methodo ipſius vix abludentem præterquam in verborum et notarum formulis* ’—“ il s'enſuivra que *ſa méthode auſſi n'eſt qu'un déguiſement de celle de M. Barrow*.”—Et non infelicitèr ſanè retorquetur à *Leibnitzio*; *Newtonum* enim potiùs mutuâſſe ab amico, ſocio, ſtudiorúmque patrono, ipſiúſque prædeceſſore in munere profeſſorio *Lucaſiano* CANTABRIGIÆ, optimo eruditiſſimóque *Barrow*; quàm *Leibnitzium* ſuam methodum ab eodem Barrow derivâſſe, ſcriptore, ſcilicet, quoad Leibnitzium peregrino et fortaſſe parùm noto, longè veriſimilius eſt.

10. Quantum verò debuit *Newtonus* anteceſſoribus ſuis, ipſe haud rarò ingenuè fatetur—modeſtiâ quàm ingenio non minùs ſpectabilis. Debuit *Sluſio* quidem in *methodo Tangentium*, imprimíſque *Barrow*, in methodo ducendi Tangentes per *triangulum arithmeticum*; *Nicolao Mercatori*, in *Logarithmotechniâ*, quam à *diviſione fractionum* ad *extractionem radicum* per *theorema binomium*, extendit Newtonus; *Hugenio*, “ in eximio ſuo tractatu *de horologio oſcillatorio*,” cujus methodum determinandi *vires centripetas et centrifugas* corporum revolventium in *circulis*, ad vires corporum revolventium in *curvis quibuſcunque*, ſolertiſſimè extendit Newtonus per *circulos ejuſdem curvaturæ*; debuit etiam *Walliſio*, in *quadraturâ curvarum*, reducendo *ordinatas irrationales* ad *rationales*, per *ſeries infinitas*; *Fermatio*, in ſolutione problematum de *maximis et minimis*; *Cavallerio*, in *methodo indiviſibilium*; atque ipſi *Carteſio*, in applicatione *Algebræ ad Geometriam*, quam ipſe tantoperè auxit, atque ad *mechanicam rationalem*, (ſeu “ *ſcientiam motuum* qui ex *viribus quibuſcunque* reſultant, et *virium* quæ ad *motus quoſcunque* requiruntur, accuratè propoſitam ac demonſtratam ”)—ſolertiſſimè extendit et accommodavit.

11. *Magnus* ſanè habendus eſt ille *inventor*, qui obſcura et involuta ſcientiæ arcana, arte ſuâ indagatrice, enucleavit et detexit, lucem ex tenebris promens; *major* tamen eſſe videbitur, ſi *ipſe expoſitor*, quæ detexit, ea *noverit* et *voluerit* certâ ac idoneâ ratione demonſtrare;—“ *analyſi breviſſimâ et ſimul perſpicuâ, ſyntheſi concinnâ et minimè operoſâ*,” qualem exercebat olìm magnus ille Geometra *Apollonius*, in libro ſuo eximio de *Sectione Rationis*, teſtante *Halley*; atque minimè deficit ab ejus veſtigiis *Newtonus*, in *Lemmate* II. *Lib.* II. *Princip.* imperatoriâ quâdam brevitate, et lucido ordine, fundamentum methodi ſuæ generalis fluxionum poſiturus.

12. Veruntamen fatendum eſt, quod *Newtonus*, nimìs forſàn brevitatis ſtudioſus et moram et moleſtiam interpretandi refugiens, non niſi “ *peritos alloquens*,” ſemina tantùm expoſitionis veræ atque genuinæ ſevit, à poſteris ſerò explicanda in

frugem uberiorem juſtæ atque legitimæ demonſtrationis, ubi "*Opinionum commenta deleverit dies*;" et tandem, poſt lapſum annorum centùm, conſenſu communi ſectatorum ſcholæ *Newtonianæ* et *Leibnitzianæ*, methodi ipſius generalis ſive *fluxionalis* ſive *differentialis*, rudimenta meliùs intelligantur, ejúſque elementa prima "*quæ tam altè jacent*," imis in penetralibus ſcientiæ *metaphyſicæ*, cautiùs atque curatiùs "*eruere*," et argumentis idoneis et ad naturam ſcientiæ illuſtrandæ accommodatis tandem confirmare, fas ſit.

13. Neutiquam cuivis homini contingit inventa ſua, à primordiis ad ultimum perfectionis ſtatum, ſpatio vitæ vel longiſſimæ perducere. Artis ſcientiaéve cujuſvis elementa ſimùl invenire, et inventa demonſtrare, vel maximis ingeniis rariſſimè conceſſit Deus. Gradatìm et ſeriatìm fit VERITATIS exploratio.—"*Plantabat Paulus, irrigabat Apollos, ſed* DEUS *augebat*"—haud minùs verum eſt in *Philoſophiâ* quam in *Religione*.

"*Felix*" quidem cenſendus eſt, "*qui potuit*" vel *paucos* annulos arripere tantùm, eoſdémque *infimos*, immenſæ illius catenæ VERITATUM ÆTERNARUM, quam à cœlis ad tellurem uſque demiſit, ægris mortalibus et hallucinantibus, benignus ILLE DEUS, qui variis gaudet nominibus—Ὁ ΛΟΓΟΣ ΤΟΥ ΘΕΟΥ, Ἡ ΣΟΦΙΑ, ſeu "RECTA RATIO," "SAPIENTIA," "VERITAS,"—tàm apud philoſophos profanos quàm apud ſacros; et qui hanc catenam non viris ingenii et ſcientiæ ſuæ opinione tumidis, ſed potiùs iis qui humiliùs de ſe ſentiunt et vitæ et morum ſanctitate inſignes fiunt, porrigit,—*paucis* quidem "*quos æquus amavit* JUPITER"—"*Divûm hominúmque æterna poteſtas*; tales verſùs ſeſe, lenitèr et indeſinentèr attrahens atque attollens:—alligatur enim *ſummus* hujúſce catenæ annulus ad baſin ipſius *throni divini*. Vid. *John* xii. 32.

Και εδικαιωθη Ἡ ΣΟΦΙΑ *απο των τεκνων αὐτης ἁπαντων*. *Luk.* vii. 35.

Licèt enim puſilli *Sophiſtæ*, ſeu *Pſeudo-Philoſophi*, *veritates æternas* haud rarò obfuſcent, ipſámque "ALMAM SAPIENTIAM" vilipendant et obtrectent ingrati; "*indies nihilominùs*," animosè vindicatur et "*juſtificatur*, *liberis ſuis omnibus*" verè philoſophantibus, à primordiis rerum; quales identidem prodierunt, "*lucida ſidera*," præclara lumina mundi: (*b*) ENOCH, NOAH, JOB, ABRAHAM, MOSES, DANIEL *Archimagus*, et PAULUS *Tarſenſis*; THALES, PYTHAGORAS, SOCRATES,

---

(*b*) 1. ENOCH antediluvianus, et "ab *Adamo* ſeptimus," tranſcendentem ob pietatem vivus tranſlatus eſt in cœlum, ætatis ſuæ anno 365mo; veros dies anni *ſolaris* mirè exponente; DEO forſàn, illi, quaſi *aſtronomo*, id revelante; ſicut illi, quaſi *prophetæ*, revelavit *ſtatum futurum*, et *judicium generale*, quæ prædixit *Enoch*, denunciavítque impiis, *Jude* xiv.—Hoc enim (licèt inficiantur *ſemi-philoſophi* et *ſemi-theologi* hodierni) dogma fuit SAPIENTIÆ PRIMÆVÆ antiquiſſimum:

> Ὁτι Ὁ ΘΕΟΣ ΕΣΤΙ, *καὶ τοις εκζητουσιν*
> *Αυτον* ΜΙΣΘΑΠΟΔΟΤΗΣ ΓΙΝΕΤΑΙ.
>
> "*Quod* DEUS EST, et exquirentibus Ipſum
> REMUNERATOR fit." *Heb.* xi. 6.

Et "*ultimis* quoque *diebus*," *τη παλιγγενεσια*,—ubi "*magnus ab integro ſæclorum naſcitur ordo*"—"*dicet vir*" omnis verè philoſophans:

Socrates, **Plato**, **Archimedes**, et **Hipparchus** *ὁ φιλαληθεστατος*; **Copernicus**, **Galileo**, **Rogerus Bacon** et **Franciscus Bacon**, **Pascal**, **Boyle**, **Barrow**, **Newton**, **Maclaurin**, **Cotes**, **Daniel Bernoulli**, &c. &c. &c. et noviſſimè quidem doctiſſimus vir, Gulielmus Jones, eques auratus, Curiæ Supremæ Britannicæ apud Calcuttam in Indiâ Juſticiarius, (Gulielmi Jones, celeberrimi Mathematici ſub initio hujus ſæculi, filius,) *Indorum Apoſtolus*: qui, ſuis in ſæculis non magìs claruerunt omnes, famâ ſcientiæ reconditioris, quam virtutis pietatíſque ſublimioris; et etiamnum, unuſquiſque eorum *αποθανων ετι λαλειται*, "*licet mortuus, adhuc laudantium in ore eſt*;" honóſque eorum in æternum vigebit.

De Objectionibus allatis contrà Methodum Fluxionum à præcipuis Auctoribus qui hactenùs impugnant ejuſdem Evidentiam et Certitudinem.

14. **Confestim** à primâ expoſitione perbrevi *Methodi Fluxionum*, novæ, ſubtilioris et reconditioris, objectiones variæ ſuboriebantur, Analyſtarum tam peritorum

---

"*Certè fructus eſt juſto:*
Certè **Deus**, *judicans in terrâ.*" *Pſal.* lviii. 11.

"*An* **Judex totius terræ** *non faciet judicium?*" *Gen.* xviii. 25.

2. **Noah**, magnus ille propheta et inſtaurator generis humani, "*qui gratiam invenit apud* **Deum**," à *diluvio* ſervatus; quod prædixit, fruſtráque denunciavit, 2 *Pet.* ii. 5. et *fortunas Noachidarum* cecinit moribundus, *Gen.* ix. 25.—Et "Inſtitutionis *Menu*" apud *Indos*, (qui *Manes* eſt dictus apud Ægyptios,) etiamnum traditione celebrantur.—*Cyclus magnus* 600 annorum, Antidiluvianus, à *Joſepho* notatus, ætatem *Noæ* tempore diluvii mirè ſervat.

3. *Job*, à Diluvio ſeptimus (Vide *Abulfaragi*, p. 13.)—*ὁ μεγας οντως εκεινος και γενναιος της αληθειας αγωνιστης*—"*magnus ille et nobilis* **veritatis** *vindex*"—*Suidas*—peritiſſimúſque *aſtronomus*, qui conſtellationes cardinales ſphæræ primitivæ recenſet, *Job* ix. 9. et xxxviii. 32. **Aish**, **Chimah**, **Chesil** et **Mazaroth**, ſive "*Urſa Major*, *Taurus*, *Scorpio* et *Canis*" reſpectivè; ut videre licet in opere quodam an. 1799 edito, cui titulus, The Inspector—*London*, *White*.

4. **Abraham**,—"*Amicus ille* **Dei**," et "*Pater fidelium*;" idémque peritiſſimus aſtronomus; ſicut per totam Orientem adhuc celebratur:—*Ασκηθεις, κατα τον πατριον νομον, τας των επουρανιων αστερων κινησεις· και στοχασαμενος ως ουκ εν τουτοις ισταται το μεγαλουργον της φαινομενης ταυτησι κτισεως, αλλ' εχει τινα τον* ΔΗΜΙΟΥΡΓΟΝ *και κινουντα και διευθυνοντα την εναρμονιον των αστερων πορειαν, και του κοσμου παντος την καταστασιν.*

"Exercitatus, patrio more *Chaldæorum*, in *aſtrorum motibus* obſervandis: atque conjecturâ aſſecutus eſt, quod non in *iis ipſis* conſiſtit *vis magna effectrix* apparentis hujúſce creationis; ſed quod potiùs habet aliquem opificem publicum, qui et movet et dirigit *harmonicum* aſtrorum curſum et mundi univerſi conſtitutionem."——*Suidas*.

5. **Moses**,—maximus ille legiſlator, "*in omni ſapientiâ* **Ægyptiorum** *inſtructus*;" qui inſtitutione *Paſchæ*, (Deo forsàn revelante,) cyclum illum *decennovalem lunæ*, celeberrimum tradidit, in computandis noviluniis, et *paſchalibus* pleniluniis, apprimè neceſſarium; cujus ope, *Meton*, longo poſt tempore, Calendarium *Athenienſe* emendavit.

6. **Daniel**

peritorum quàm imperitorum, validitatem ejus denegantium; ac, (non obſtantibus magnis illis Newtoni diſcipulis,—COLSON, qui anno 1736, *Newtoni Methodum Fluxionum Analyſi Æquationum per ſeries infinitas intertextam*, verſione Anglicâ et commentario copioſo donavit et illuſtravit;—MACLAURIN, qui anno 1742, *Tractatum* ſuum *Fluxionum*, Anglicè edidit, fundamentum totius methodi generalis à primordiis ad mentem *Newtoni* in *Lemmate Secundo*, Lib. II. Princip. explicare profeſſus;—et STEWART, qui anno 1745, duo ipſius Newtoni celeberrima opera, *Analyſin Æquationum per ſeries numero terminorum infinitas*, et *Quadraturam Curvarum*, verſione Anglicâ et commentario uberrimo inſtruxit;—aliiſque ſcholæ *Newtonianæ* diſcipulis celeberrimis, *Simpſon*, *Saunderſon*, *Emerſon*, *Waring*, &c.) adhuc vigent objectiones tàm noſtratium *Berkeley*, *Landen*, &c. ac nuperrimè ſcriptoris doctiſſimi in libro cenſorio, ſingulis menſibus edito, cui titulus, *The Monthly Review*; quàm exterorum *D'Alembert*, *Torelli*, *La Grange*, &c. optimorum, ſcilicet, expoſitorum *Methodi Differentialis*, et acerrimorum obtrectatorum *Methodi Fluxionum*.

Et certè fatendum eſt, non obſtantibus tot commentariis tantorum Analyſtarum, qui fines Fluxionum magnoperè extenderunt, hactenùs deſiderari ipſius *Textûs Newtoniani*, (breviſſimi ac reconditiſſimi,) interpretem fidum atque perſpicuum, methodi et lucidi ordinis ab ipſo auctore inſtituti ſtrenuè ſequacem, et expoſitione nec nimis curtâ, nec nimis prolixâ, difficilia illuſtrantem; (vitia, quæ utcunque ſibi mutuò contraria videantur, tamen haud rarò ſimùl et ſemèl inveniuntur;) ac de ſenſu nativo ipſius authoris eruendo, quàm de ſuâ eruditione aut facundiâ oſtentandâ, magìs ſollicitum.

6. DANIEL ille *Archimagus*, in omni ſapientiâ et diſciplinâ *Magorum* et *Chaldæorum* inſtructus, atque ita celebratus ut "*ſapientior Daniele*" in proverbium eſt evectum, per totam Orientem, *Ezek.* xxviii. 3.—Et in Sacris Scripturis honoratiſſimè diſtinguuntur—"*Noah*, *Daniel* et *Job*"—quaſi præpotentes *interceſſores* cum DEO—*Ezek.* xiv. 14.—*Noah*, pro ſe ſuâque familiâ, liberatis ab excidio diluvii—*Daniel*, pro reditu *Judæorum* Captivorum;—et *Job*, pro amicis ſuis temerè philoſophantibus, et PROVIDENTIAM DIVINAM ignavè vindicantibus.

7. PAULUS *Tarſenſis*—"apud pedes *Gamalielis*, celeberrimi præceptoris *Judæorum*, educatus;" *Acts* xxii. 3. et in omni diſciplinâ ſcholarum philoſophicarum *Alexandriæ*, *Athenis* et *Romæ*, non levitèr inſtructus.

8. THALES, eclipſin illam ſolarem prædixit, quæ diremit pugnam inter *Lydios Medòſque* commiſſam, *Julii* 30, *anno* 607, *ante æram Chriſti vulgarem*, (ut poſthac forſàn probare mihi dabitur occaſio in *diſſertatione* quâdam *De Chronologiâ Antiquâ*, mox edendâ—ſi DEUS *det vitam, det opes*)—epochâ, nimirùm, ab aſtronomis chronologiſve hactenùs non ſatìs notatâ. Hanc verò eclipſin, tantoperè litigatam, facilè computare potuiſſet *Thales*, ope *Cycli* illius *Decennovalis*, ex *Judæis Chaldæiſve* mutuati—à *Græcis* et *Mètone*—qui accuratiùs conſtabat ex *lunationibus* 223, ſpatio *annorum* 18 *cum diebus* 11, peractis—quem figuratè et aptè innuit *Theocritus*, *Idyll.* XV.

Ὀκτωκαιδεκαέτης ἢ ἐννεακαιδεκ' ὁ γαμβρὸς——

De nuptiis Adonidis loquens, ſeu *conjunctionibus Solis et Lunæ*—"*octodecimo vel nonodecimo anno.*"

9. PYTHAGORAS, ille *Samius*, cujus allegoriam eximiam de ſemitis vitæ divergentibus ad *veritatem* vel ad *errorem*, per furculas literæ Υ, deſignatis, notat *Perſius*, *Sat.* III. 56.

Et tibi quæ SAMIOS *deduxit litera ramos,*
Surgentem DEXTRO *monſtravit limite callem.*

Quam lucubentiſſimè exponere non dedignatus eſt ipſe JESUS CHRISTUS, *Matt.* vii. 13.—Verum quoque mundi ſyſtema, ſeu *Copernicanum*, aut detexiſſe, aut ſaltèm ex *Chaldæis* didiciſſe, videtur.—Vide *Maclaurin*'s *Account*, *Chap.* 2.

15. Audiamus THE MONTHLY REVIEW, in eruditâ admodùm recenſione ſuâ libri cui titulus *Theorie des Fonctions Analytiques*, par *La Grange*, *Paris* 1798. *Art.* 1. *Append.* Vol. 28.

“ *The Method of Fluxions* had never ſo acute, ſo learned, and ſo judicious a defender as *Maclaurin.*—His work is moſt valuable for the variety of information which it contains, and for many ſpecies of clear and forcible argumentation: yet whoever conſults it, and ſees the various *artifices* which he is obliged to employ for the purpoſe of demonſtrating the ſeveral parts of the method; and the grounds of it not eſtabliſhed in leſs than *fifteen long theorems*; and, in an enquiry *purely mathematical*, finds the author ſpeaking of “ cauſes and effects,” “ of the ſprings and principles of things,” and propoſing to deduce “ the relations of quantities by comparing the powers which are conceived to generate them;”—will be convinced that this could only happen from ſo able a mathematician having failed to ſeize *the right principle* and the *true mode of explanation.*”

16. Atque hujus acutiſſimi cenſoris ſententiæ multis nominibus accedo: nam, primò, à verâ interpretandi viâ deflectens, *Maclaurinus* illuſtrationibus geometricis, à naturâ *motûs* petitis, nimìs immoratur in libro primo; longo ſermone luxurians, et argumentationis ambagibus ad faſtidium uſque indulgens, à genio *Newtoniano* prorsùs alienis; qui *methodum ſuam Fluxionum* excogitavit, “ *ut effugeret tædium deducendi longas demonſtrationes, more veterum geometrarum, ad abſurdum*;”—et “ *contractiores*” his, et ſimùl accuratiores illis quæ per *methodum indiviſibilium* à Cavallerio ejúſque diſcipulis eliciebantur, magno ſcientiarum emolumento, ſubſtituit. Contrà, dum brevis eſſe laborat, obſcurus fit *Maclaurinus* in libro ſecundo; et, quaſi interpretandi pertæſus, primo hujus capite, ſtrictìm tantùm, principiis veris ac genuinis methodi univerſalis, (ex naturâ *relationum* quantitatum variabilium abſtractè conſideratarum, manantibus,) vix, aut levitèr tantùm notatis, *elementa prima* ſeu *metaphyſica*, à *Newtono* poſita, feſtinantèr tranſcurrit; et *principia mathematica* quoque nimiâ et præproperâ celeritate abſolvit; et denique, ipſius inventoris lucidum ordinem ſtatìm ab initio deſerens, multa mutat; et quæ mutat, corrumpit: nam,

2. *Prop.* 1. Totius theoriæ ſuæ fundamentalem, § 707, “ *fluxionem quadrati* A^2, *æquari* $2aA$;” non niſi imperfectè demonſtrat; unde enim hanc fluxionem *aſſumpſerat* (*c*), minimè oſtendens, tranſit, per ſaltum, ad demonſtrandam veram fluxionem hâc, nec *majorem* eſſe, nec *minorem*, argumento ad abſurdum prolixiore uſus.

(*c*) The defect of *Maclaurin*'s demonſtration may thus be ſupplied, from his own principles rightly conceived, though rather confuſedly expreſſed, § 701—706.

§ 707. *Prop.* 1. *The fluxion of the root* A *being ſuppoſed equal to* a, *the fluxion of the ſquare* AA *will be equal to* 2aA.

Let $A - a$, A, and $A + a$, be ſucceſſive values of the variable root; and the correſponding values of the variable ſquare will be $A^2 - 2aA + a^2$, A^2, and $A^2 + 2aA + a^2$; of which the firſt

uſus. Et inde, § 708, "*fluxionem rectanguli* A + B *æquari* $aB + bA$," pro corollario, deducit. At artifex ipſe, longè elegantiùs atque peritiùs, auſpicando à *caſu primo*, et generaliore,—*fluxione* nempe *rectanguli*, inde particularem,—*fluxionem* nempe *quadrati*,—facillimè deducit, *Caſ.* 3.

3. Per varios anfractus procedens, *lemmata* compluria, ad demonſtrandas propoſitiones inſequentes, tortuosè introducit *Maclaurinus*, à quibus immunis erat *Methodus Newtoniana*.

4. Ad methodum inventoris jam ſerò reverſus, quam adeò infelicitèr initio deſeruerat, breviùs et meliùs demonſtrat "*fluxionem* A^n *æquari* naA^{n-1}," more *Newtoniano*, § 716, quam imperfectè et prolixè demonſtraverat anteà, § 712, ſecutus *Prop.* 1.

5. Negligentiâ minimè condonandâ, *Theorematis illius generaliſſimi*, *Lem.* II. *Lib.* II. *Princip.* *ultimum caſum*, quem à præcedentibus, directo atque immediato nexu, ſummo acumine, ſummâque peritiâ Algebraicâ, deduxerat *Newtonus*, nempe, *fluxionem* $A^m B^n$ *æquari* $maB^n A^{m-1} + nbA^m B^{n-1}$, prorſùs omittit; itémque *tria corollaria* inſequentia; elegantiſſima atque uberrima quidem, quæ calculo fluxionali concinnando, ubi de *proportionalibus* agitur, et de *logarithmis* præſertìm, magnoperè inſerviunt, eâdem negligentiâ omiſit.

is the leaſt, and the laſt the greateſt. From this laſt value of the ſquare ſubtract the firſt, and, the extreme terms A^2 and a^2 being deſtroyed, there will remain the *exceſs* $+ 4aA$; which will denote the *whole* increaſe of the ſquare from the firſt $(A - a)^2$ to the laſt $(A + a)^2$, with the increment a, *twice*, or $2a$; (which is the difference between the extreme values of the root, $A - a$ and $A + a$); and conſequently *half the exceſs*, namely $2aA$, will denote the increaſe with the increment a *once*; or will *meaſure the rate of variation*, or be *the fluxion* of the variable ſquare, from its *leaſt* to its *mean* ſtate AA.

If you deny it, let the true fluxion be either greater or leſs than $2aA$, by any aſſignable difference D, or equal to $2aA \pm D$. But if greater, the laſt increment muſt neceſſarily be greater than $+ 2aA + a^2$; if leſs, the firſt increment muſt be leſs than $+ 2aA - a^2$ (or the difference between the firſt and mean values of the variable ſquare), contrary to the hypotheſis. Q. E. D.

In numbers the caſe is plain:

Let *the root* $A = 10$; and its *momentary increment* $a = 1$.

$$\begin{array}{llc}
 & \textit{Val.} & \textit{Diff.} \\
(A - a) \times (A - a) = & 9 \times 9 = 81. & \multirow{2}{*}{$\left.\right\} 19$} \\
A \times A \text{ - - - - - - - - } = & 10 \times 10 = 100 & \\
(A + a) \times (A + a) = & 11 \times 11 = 121 & \left.\right\} \frac{21}{40}. \\
\text{But - - } 4aA \text{ - - - } = & 4 \times 10 = (40 =)\ 19 + 21. & \\
\text{Therefore } 2aA \text{ - - - } = & 2 \times 10 = (20 =)\ \frac{19 + 21}{2}. &
\end{array}$$

Conſequently $2aA$ expreſſes the *fluxion* or *rate* of increaſe of the variable ſquare in its advance to the *mean* ſtate $A \times A$.—The demonſtration might be abridged upon *Newton*'s principles, by taking only the half increment:—For the leaſt value $(A - \frac{1}{2}a)^2$ becomes $A^2 - aA + \frac{1}{4}a^2$, and the greateſt, $(A + \frac{1}{2}a)^2 = A^2 + aA + \frac{1}{4}a^2$. But the exceſs, $+ 2aA$, gives *the fluxion* at once, during the *whole* momentary increaſe. See § 84 and 85.

17. Diluendis igitùr objectionibus eorum qui methodi fluxionum generalis evidentiam atque certitudinem impugnant, omninò imparem eſſe commentarium *Maclaurini*, quamvis valdè eruditi, et in hiſce ſcientiis verſatiſſimi, et (quod auctoritatem majorem ipſi parere debere ſuper hoc argumento videatur,) in ipſius *Newtoni* amicitiam (*d*) admiſſi, quis mathematicus non videt?—At non levi detrimento theoriæ fluxionum, iſte commentarius fuſior, qualis qualis fuerit, mox ſupplantavit originalem textum, paullò ſubtiliorem atque difficiliorem intellectu, ob *imperatoriam* auctoris *brevitatem*, non niſi auribus rite purgatis capiendam—

Φωναντα συνετοισι.

Ex hoc fonte igitùr profluxerunt vel perſpicaciorum peritiorúmque analyſtarum objectiones, ſcholæ *Leibnitzianæ* præſertìm, adverſùs ipſam fluxionum methodum; quaſi methodo *differentiali*, evidentiâ et certitudine, cedentem, et in principiis à geometriâ alienis fundatam.

18. Audiamus *D'Alembert*, analyſtam adeò celebratum, in *Matheſi* perſpicaciſſimum, *in religione* hebetiſſimum, argumento paulò ſubtiliore validitatem exponendi *fluxiones* per velocitates, impugnantem:

" Introduire ici *le mouvement*, c'eſt y introduire une idée *étrangère*, et qui n'eſt point *néceſſaire* à la démonſtration: d'ailleurs, on n'a pas d'idée bien nette de ce que c'eſt *la vîteſſe* d'un corps à chaque inſtant, lorſque cette vîteſſe eſt variable. La vîteſſe n'eſt rien de *réel*, c'eſt le *rapport* de l'*eſpace* au *tems*, lorſque la vîteſſe eſt *uniforme*; mais lorſque le mouvement eſt variable, ce n'eſt plus le rapport de l'eſpace au tems, c'eſt le rapport de *la differentielle* de l'eſpace à cette

---

(*d*) The following letters of *Newton* reflect high honour on his liberality of mind and delicacy of friendſhip towards the excellent *Maclaurin*, when a candidate for the joint *Profeſſorſhip of Mathematics* in the Univerſity of *Edinburgh*.—In one of theſe letters, addreſſed to Mr. *Maclaurin*, (with permiſſion to ſhew it to the Patrons of the Univerſity,) Sir *Iſaac* expreſſes himſelf thus:

" I am very glad to hear that you have a proſpect of being joined to Mr. *James Gregory* in the Profeſſorſhip of the Mathematics at *Edinburgh*, not only *becauſe you are my friend*, but principally *becauſe of your abilities*; you being acquainted as well with the *new improvements* of Mathematics, as with the *former ſtate* of thoſe ſciences: I heartily wiſh you good ſucceſs, and ſhall be very glad of hearing of your being elected. I am, with all ſincerity, your faithful friend and moſt humble ſervant."

In a ſecond letter, which was addreſſed to the then Lord Provoſt of *Edinburgh*, (which Mr. *Maclaurin* knew nothing of till ſome years after Sir *Iſaac*'s death,) he thus writes:

" I am glad to underſtand, that Mr. *Maclaurin* is in good repute amongſt you for his ſkill in Mathematics; for *I think he deſerves it very well*: And, to ſatisfy you that I do not flatter him, and alſo to encourage him to accept the place of aſſiſting Mr. *Gregory*, in order to ſucceed him, I am ready (if you pleaſe to give me leave,) to contribute *twenty pounds per annum* towards a proviſion for him, till Mr. Gregory's place becomes vacant, if I live ſo long, and I will pay it to his order in *London*."

See an *Account of the Life and Writings of Maclaurin*, prefixed to his *Account of Sir Iſaac Newton's Philoſophical Diſcoveries*, in which are many curious and valuable particulars of the life of that CHRISTIAN PHILOSOPHER, eſpecially in his laſt moments; ſo different from thoſe of that *miſerable* infidel *D'Alembert*.——APPENDIX II. § 161 & 164.

du tems; rapport dont on ne peut donner d'idée nette que par celle des *limites*. Ainſi *il faut néceſſairement en revenir à cette dernière idée, pour donner une idée nette des fluxions.*"

19. Quinquaginta retrò annis circitèr, talia objecit vir clariſſimus *D'Alembert*, cujus auctoritas nunc tanti æſtimatur. Ac nuperrimè quidem, ("*vires* enim *acquirunt eundo*,") maximus ille promotor Algebræ puræ, nec non analyſeos ſublimioris, *De La Grange*, in opere quodam elementari, cui titulus, THEORIE DES FONCTIONS ANALYTIQUES, 1798, *Paris*, autumat ſe ſerò tandem vera et genuina principia *calculi differentialis* extudiſſe, ab omni conſideratione ſive *quantitatum infinitè parvarum*, ſeu *evaneſcentium*, ſive *limitum* vel *fluxionum*, libera atque immunia; uſurpando tantùm *Quantitatum finitarum Analyſin Algebräicam*.

20. Hiſce magni nominis viris conſentiens *Landen* noſtras, ita doctrinam fluxionum perſtringit in *Analyſi ſuâ reſiduâ*, anno 1758—Analyſta et ipſe ex peritiſſimis.

"It ſeems more proper, in the inveſtigation of propoſitions by *Algebra*, to proceed upon the *anciently-received principles* of that art, than to introduce therein, *without any neceſſity*, the *new* fluxionary principles derived from a conſideration of *motion:* and the rather, as the introduction of theſe new principles is not attended with any *peculiar* advantage."

21. Tantis opponentibus accedit quartus,—nimirùm *cenſor* ille, in hiſce ſcientiis non levitèr verſatus, (quiſquis fuerit,) apud THE MONTHLY REVIEW, *Append.* Vol. 28, 1799,—ex cathedrâ moderatoris, ita pronuncians judicium, in *Newtonum* ejúſque commentatores et diſcipulos:

"Of the *Method of Fluxions*, properly ſo called, *Newton* is the ſole and undiſputed inventor; to him, beyond all doubt and controverſy, is to be attributed the *eſtabliſhment* of that doctrine on the conſideration of *Motion*.

"Had not *Newton* been the inventor of *this* method of conſidering Fluxions, WE are of opinion, that very few perſons would have contended either for its *metaphyſical* or *mathematical* excellence: for, on the ground of perſpicuity and evidence, the underſtanding is not much aſſiſted by being directed to conſider *all* quantities as generated by *Motion*. *Lines* we can conceive generated by the motion of *points*; *ſurfaces*, by the motion of *lines*; &c. But when ſuch quantities as *Weight*, *Denſity*, *Force*, *Reſiſtance*, &c. become the objects of enquiry, and are ſaid to have their *flux* and *velocity*, then the true end of the *figurative* mode of ſpeech—*Illuſtration*—is loſt: On the ground of *mathematical* convenience—that the definition of Fluxions, as given, is commodious to deduce thence their properties and affections—nothing is gained; ſince the enquiry is reduced "*ultimately*" (*en dernière analyſe*) to the determination of *the ultimate ratio* of vaniſhing quantities, or of *the limits* of ſuch quantities: which latter method is only the former, *algebräically* expreſſed.

"That which happened to *Ariſtotle* has happened to *Newton:* His followers have bowed ſo implicitly to his *authority*, that they have not exerciſed their *reaſon*.—If the *Engliſh* Mathematicians firſt adopted *Newton*'s Method from veneration to him, or *for want of better information*, they have ſince perſevered in it, (WE may almoſt ſay) *againſt conviction*.—

"It has been ſaid, that *the true principles of any invention are moſt ſatisfactorily and truly ſtated by the inventors themſelves*;—yet it is at the diſtance of more than a hundred years from the diſcovery of the *Fluxionary Calculus*, that its principles are firſt clearly and rigorouſly eſtabliſhed. It has been reſerved for the Mathematicians of *theſe days*, to effect what was denied to a *Newton* and a *Leibnitz*; and to ſupply to their theory that evidence and exactneſs, the want of which makes us attribute its invention rather to the felicity of the times in which they lived, than to the excellence of their genius:—to eſteem it, not *pro partu ingenii*, but *pro partu temporis*."

---

22. Eruditorum ſanè graves ſunt hæ objectiones, et graviter pronunciatæ. Veruntamen, non obſtantibus tantis judicibus, et poſthabitâ omni *auctoritate*, (quæ in MATHESI minimè valere debet,) fidentèr affirmare auſim, propriis ex inveſtigationibus, *Nullius addictus jurare in verba magiſtri*—à nemine, ne vel ab ipſo *De La Grange*,—evidentiùs, accuratiùs, ſolidiùs, invictiùs et generaliùs, ſtrui fundamentum totius *Theoriæ Fluxionum*, quàm à celeberrimo inventore, *ſagaciſſimi ingenii* viro; ſi modò, miſſis aliorum commentis, adeamus ipſius *textum originalem*; nam, ſi attentè et circumſpectè hic textus inſpiciatur, et, contextu examinato, cæteriſque auctoris documentis criticè collatis, probè intelligatur, terminis technicis aptè definitis, atque ellipſibus ritè ſuppletis, (quæ cauſæ ſunt ex quibus præcipuæ difficultates in Newtoni ſcriptis oriuntur) cenſebit, nì fallor, omnis æquus peritúſque controverſiæ hujus judex, objectiones hactenùs prolatas, vel graviſſimas, ex inſcitiâ potiùs, ex imperitiâ, præjudicio, vel livore, quàm ex juſtis ratiociniis et veritatis perceptione ortas eſſe; imò in ipſos objectores jure retorqueri poſſe.

23. Analyſtas *exteros* quidem, eoſdémque *Gallos*, *nomini Britannico* olìm infenſos et nunc acerbiſſimè,—*D'Alembert* aut *De La Grange* (e)—ipſius *Newtoni* æmulos minimè impares, de laude ejus detrectare voluiſſe, non eſt mirum; qui nec

---

(e) DE LA GRANGE.—Quindecim retrò annis, hæc notabam fidus NEWTONI diſcipulus. ANALYS. ÆQUAT. § 259.

"Ex dictis facilè judicabit lector, an valent multi nominis viri *La Grange* objectiones. *Diſſert. Acad. Berlin*, an. 1768: nempè quod methodo *Newtonianâ* [extrahendi radices æquationum quàm proximè, in *Analyſi Æquationum per ſeries infinitas*, quàm demonſtratione generali munieram § 249—259.] ignoratur quantum ad veritatem accedit tùm valor (k) primò aſſumptus, tùm valor cujuſque complementi inſequentis (p), (q), &c.—Methodus certè quam ut accuratiorem commendat, *Newtonianæ* facilitate longè cedit; quippe quæ ex inventione quantitatis $\sqrt{1}$, § 234, non niſi per *tentamina*, nec raró nec parcè, quamvis furtìm, inventiones ejus reconditiſſimas enucleare compotes, ad ſe ſuóſque tranſtulerunt: Cæterùm híſce temerè conſentire Analyſtas *Britannicos*, auctoritati alienæ ignavè ſuccumbentes, pudet dolétque. Horum ſanè adſcribi ordinibus *Landen* patiamur forsàn, qui ſuis ſtudiis, ſuíſque opinionibus nimìs et pertinacitèr erat addictus, ita quòd *Analyſin* ſuam *reſidualem*, (quâ tantoperè delectabatur,) ipſi *Methodo Fluxionum*, (cujus larva tantùm habenda ſit, ut mox oſtenditur,) arrogantèr et falſò antepoſuerit; et qui, cum Analyſtæ, hujus ævi, celeberrimi, *D'Alembert* et *Euler*, adſtipulantibus *Wildbore* et *Friſi*, communi conſenſu, ex diverſis principiis, determinârant eandem veram quantitatem *motûs corporis circà axem liberè rotantis, ubi actione vis cujuſdam extraneæ, aut percutientis aut accelerantis, perturbatur*—ſuam concluſionem, ab eorum concluſione non modicè diſſentientem, eorum auctoritati quadruplici opponere ad mortem uſque non deſiit; pertinaciæ exemplum præbens, quòd maximis ingeniis haud raró contingit, plus æquo "*ſibi fidentibus aliáque ſpernentibus:*" At verò *cenſorem publicum*, apud M. R.; ipſúmque, ut ferunt, *Profeſſorem Matheſeos Academicum*, ita peccâſſe, minimè condonandum eſt. Valdè equidem ſuſpicor, illum, λογῳ αργῳ, "*ignavâ ratione*" ſeu negligentiâ, *ex nimiâ et præproperâ mentis feſtinatione*, hæc opinionum commenta

tentamina, valdè laborioſa in æquationibus altiorum demenſionum, eruendæ, auſpicatur; per inventionem *Limitum Æquationum* ex binomiis compluribus ſucceſſivè ſubſtituendis, procedit; et per combinationem plurium *fractionum continuarum*, ex his operationibus elicitarum, concludit."

Quantâ autem comitate, quanto animi candore ſuſtinuit *Vindicias* háſce *Newtonianas*, haùd temerè excitas, animadverſionéſque certè non leves; ex inſequenti patebit epiſtolâ, quam mihi, homini obſcuro, poſt acceptam *Analyſin Æquationum*, ſcribere non dedignatus eſt vir clariſſimus. Hanc verò, in ſcriniis meis hactenùs latentem, edere jam libet, minimè jactantiæ gratiâ,—ſtudiis meis diù converſis, Mathesi relictâ, in Musas Hæbræas—"*quarum ſacra fero, ingenti perculſus amore*"—"ſmit with the love of *ſacred ſong*"—ſed quâ par eſt obſervantiâ; ne cuívis *Vindicias ſecundas* legenti naſcatur ſuſpicio, me, animo utcunque infenſo, ſimultate quâvis, aut odio hoſtili, denegare velle grata "*præmia laudi*" tam inſigni: quem ex peritiſſimis Newtoni ſucceſſoribus, (excepto forſan *La Place*) maximíſque *Algebræ puræ* et *Analyſeos ſublimioris* hoc ævo cultoribus, facilè principem, concedere cogor *Britannus*, et non inficiabitur æqua poſteritas:

Monsieur, *Berlin, Nov.* 1. 1785.

Agréez mes très ſincères remerciemens de l'honneur que vous m'avez fait en m'envoyant votre Analyse des Æquations; *que j'ai lue avec toute la ſatisfaction poſſible. La clarté et la préciſion qui regnent dans cet ouvrage, et la réunion qu'il préſente des* METHODES ELEMENTAIRES *et des* THEORIES SUBLIMES *doivent lui donner un des premiers rangs parmi ceux de ſon genre; et par l'intéreſt que je prends aux progrès de la* Geometrie, *je partage la reconnoiſſance que vos* COMPATRIOTES *vous doivent pour ce travail. Je deſirerois fort pouvoir mériter l'opinion avantageuſe que vous voulez bien avoir de moi, mais je ne puis y répondre, que par la véritable eſtimé, et la conſidération diſtinguée avec laquelle, J'ai l'honneur d'être,*

Monsieur,

Votre très humble et très obéiſſant ſerviteur,

De la Grange.

Neque exiſtit ratio quævis, cur (in Mathematicis ſaltèm) animo infenſo judiciis cenſorum eruditorum M. R. meum opponerem, qui ſatìs ſupérque laudabant Analysin Æquationum in recenſione tractatûs illius, eodem circitèr anno; ſed cum ambitioſè in lucem proferuntur *Exterorum* opera Mathematica—Curſus Mathematici *Sauri* et *Bezart*; Inſtitutiones Analyticæ *Euleri* et *Clairaut*, *De La Place* et *De La Grange*; at *Noſtratium*, faſtidioſè in umbram relegantur, iniquâ forſan comparatione, à cenſoribus *Britannis*; cur non mihi æquo ſaltèm jure liceret viciſsìm, afferre tanti cenſoris *Galli* teſtimonium, de *unâ* ſaltèm inſtitutione elementariâ—etiàm *Hibernâ*?

ex aliis potiùs temerè aſſumpſiſſe : ídque vel ex meâ ipſius fortunâ in hâc re veriſimile mihi videtur, cui contigit adoleſcenti *Rudimenta Fluxionum* ex *Maclaurin's Treatiſe of Fluxions* didiciſſe ; et qui non niſi anno ſuperiore (1798), tandem, cùm fortè curioſiùs ipſum textum *Newtonianum* inſpexiſſem, atque ipſius auctoris ſimplicitatem elegantiámque demonſtrandi, cum commentatoris prolixitate et ambagibus, contuliſſem, ſententiam ſuper hâc re mutare ſum coactus, et à commentatoris partibus in ipſius auctoris partes tranſire, et cum admiratione exclamare,

> *Homo homini quid præſtet !*—Inventor
> Interpreti quanto antecellit !

24. Et mecum quidem et ſentit et loquitur *Colſon*, (Methodi Fluxionum *non ſordidus auctor*) híſce verbis :

"The principles of the method here taught have been *ſcrupulouſly examined* and ſifted, have been *vigorouſly oppoſed*, and, we may ſay, *ignorantly rejected* as inſufficient, by *ſome* mathematical gentlemen, [*Berkeley*, &c.] who ſeem not to have derived their knowledge of them from *their only true ſource* ; that is, our *author's own treatiſe*, written expreſsly to explain them."

25. Placet igitùr, reſuſcitatis hodiè objectionibus obſoletis, novíſque ingruentibus, et per *Angliam*, orbémque literarum, à *Galliâ* graſſantibus, totam methodum fluxionum ab ipſis fontibus, tàm *metaphyſicis* quàm *mathematicis*, ab integro ad examen rigidum revocare ; ut, his perſpectis et per ſcrutinium inveſtigatis, ne diutiùs falſis laboret criminibus evidentia ejus et certitudo ; neque *injurioſo pede proruant ſtantem columnam laudis* NEWTONIANÆ, vel imperiti vel invidi, ſublimem et ſubtiliſſimam methodum ex ſacris fontibus ANALYSEOS ANTIQUÆ hauſtam promotámque, quaſi "*partum temporis*" abortivum, ex elementis incongruis conflatum, atque caducum, inſimulantes, et non potiùs "*partum ingenii*," divinitùs conceptum, atque ad ſtatum maturæ maturum atque perfectum, plenitudine temporis in lucem editum, *hæredem immortalitatis*, venerantes.

26. Nec minùs urget impellítque ſolicitudo, curáque non levis, NE QUID DETRIMENTI CAPIAT RESPUBLICA—influxu hoc infauſto *opinionum Gallicarum*, tanto ſtudio, tantíſque illecebris, tantóque auctoritatis pondere, PHILOSOPHIAM BRITANNICAM invadentium ; et niſi maturè et ſtrenuè et peritè, "*zelo ſecundùm ſcientiam*" repellantur animosè, et rejiciantur indignantèr, IPSAM unà cum RELIGIONE ILLA, (cui modeſtè et ſobriè hactenùs ancillari ſolita eſt) ſub erroris et impietatis fluctibus obruere minitantium.

ESTO PERPETUA !
STATE SUPER VIAS ANTIQUAS !
PRO FIDE SANCTIS COMMISSA, CONTENDITE !

27. Nec operam JUVENTUTI STUDIOSÆ, (noſtrati potiſſimùm) ingratam aut inutilem me præſtiturum ſpero, ſi ſtudia *Mathematica*, jamdudùm intermiſſa, recolens, SUBSIDIA QUÆDAM ELEMENTARIA ad *Phyſicam Newtonianam* (et præſertìm *Aſtronomiam*) PHILOSOPHIÆ PRIMÆ, SACRÍSQUE LITERIS, optimè conſentientem, pro virili, brevitèr, fidelitèr, et tamen ſimplicitèr perſpicuéque, proponere aggrediar:

"To illuſtrate *Newton* is to cultivate *real ſcience*; and to make his diſcoveries eaſy and familiar, will be no ſmall improvement in MATHEMATICS and PHILOSOPHY."——*Colſon*.

De Methodo Rationum Primarum et Ultimarum, ſeu Limitum.

28. TOTIUS Methodi Fluxionum fons atque origo eſt doctrina *Rationum Primarum et Ultimarum*, ſi Geometricè loquamur, ſeu *Limitum*, ſi Algebräicè; ut rectè perhibent *D'Alembert* et *Cenſor* ille *profeſſorius*, Newtonum ipſum vel ſecuti, vel cum eo (inſcii forsàn,) conſentientes; qui ita loquitur:

——"Præmiſi verò *hæc Lemmata*, ut effugerem *tædium* deducendi longas demonſtrationes, more veterum Geometrarum, *ad abſurdum*. Contractiores enim redduntur demonſtrationes per *methodum indiviſibilium*: ſed quoniam durior eſt indiviſibilium hypotheſis, et propterea methodus illa *minùs geometrica* cenſetur, malui demonſtrationes rerum ſequentium ad *ultimas* quantitatum evaneſcentium *ſummas* et *rationes*, *primáſque* naſcentium; id eſt, ad *limites* ſummarum et rationum deducere; et propterea limitum illorum *demonſtrationes*, quâ potui brevitate, præmittere: his enim idem præſtatur quòd per methodum indiviſibilium; et *principiis demonſtratis jam tutiùs utemur*."—"*Vímque* talium demonſtrationum ad methodum *præcedentium lemmatum* ſemper revocavi."

29. Pendet ſanè omnis hæc *Methodus Limitum*, ex *Lemmate primo*, *Lib.* I. *Princip.*

"QUANTITATES, *ut et* QUANTITATUM RATIONES *quæ ad æqualitatem tempore quovis finito conſtantèr tendunt*; *et ante finem temporis illius propiùs ad invicèm accedunt quàm pro datâ quâvis differentiâ*; *fiunt* ULTIMO ÆQUALES."

"Si negas, fiant *ultimò inæquales*; et ſit earum ultima differentia D: Ergò, nequeunt *propiùs ad æqualitatem* accedere, quàm pro datâ differentiâ D: Contrà hypotheſin." Q. E. D.

Ultimæ rationes igitùr, ipſo definiente *Newtono*, et quàm accuratiſſimè, et diſtinctiſſimè, ſunt “ *limites, ad quos quantitatum ſine limite decreſcentium rationes,* 1. *ſemper appropinquant*; et 2, *quas propiùs aſſequi poſſunt quàm pro datâ quâvis differentiâ*; 3. *nunquàm verò tranſgredi*; 4. *neque priùs attingere quàm quantitates diminuuntur vel augentur in infinitum.*”

30. Sedulò autem et ſolicitè diſtinguit *Newtonus* inter harum quantitatum *magnitudines ultimas*, et *rationes ultimas.*—“ In ſequentibus igitùr ſi quando facili rerum conceptui conſulens, dixero *quantitates quàm minimas,*” vel “ *evaneſcentes* ” vel “ *ultimas,*” cave intelligas quantitates *magnitudine determinatas* ſed cogita ſemper diminuendas *ſine limite.*”—Atque iterùm :—ſi quando quantitates tanquam *ex particulis conſtantes* conſideravero, vel ſi pro *rectis* uſurpavero *lineolas curvas*; nolim *indiviſibilia* ſed evaneſcentia *diviſibilia*, non ſummas et rationes partium *determinatarum*, ſed ſummarum et rationum *limites* intelligi.

31. Nec minùs ſolicitè diluit *Newtonus* objectiones impugnantium evidentiam et certitudinem doctrinæ limitum.

Objectio eſt “ *Analyſtæ* ” (*Berkley*), quòd “ *quantitatum evaneſcentium nulla ſit ultima proportio*; quippe quæ *antequam* evanuerunt non eſt ultima; *ubi* evanuerunt, nulla eſt.”—Sed et eodem argumento, regerit *Newtonus*, æquè contendi poſſet “ *nullam eſſe corporis ad certum locum ubi motus finiatur pervenientis, velocitatem ultimam:* hanc enim, *antequam* corpus attingit locum, non eſſe ultimam; *ubi* attingit, nullum eſſe.” At reſponſio facilis eſt : Per “ *velocitatem ultimam* ” intelligi, eam, quâ corpus movetur—neque antequam attingit locum ultimum et motus ceſſat, neque poſteà—ſed tunc, *cum* attingit ; id eſt, illam ipſam velocitatem *quâcum* corpus attingit locum ultimum, et *quâcum* motus ceſſat. Et ſimilitèr, per “ *ultimam rationem quantitatum evaneſcentium,*” intelligendam eſſe rationem quantitatum, non antequam evaneſcunt, non poſteà, ſed *quâcum* evaneſcunt. Paritèr et *ratio prima naſcentium*, eſt ratio *quâcum* naſcuntur. Et *ſumma prima et ultima*, eſt *quâcum eſſe* (vel *augeri* aut *minui*) incipiunt et ceſſant. Extat *limes* quem velocitas in fine motûs *attingere* poteſt, non autem *tranſgredi*; hæc eſt velocitas ultima : Et par eſt ratio *limitis quantitatum* et *proportionum omnium* incipientium et ceſſantium. Cùm hic limes ſit certus et definitus, problema eſt *verè geometricum* eundem determinare. *Geometrica verò omnia in aliis Geometricis determinandis ac demonſtrandis, legitimè uſurpantur.*

32. “ Contendi etiàm poteſt, quòd “ *ſi dentur ultimæ quantitatum evaneſcentium rationes, dabuntur et ipſæ magnitudines ultimæ.*” Hæc verò conſequentia non eſt neceſſaria. Nam fieri poteſt ut claram ideam habere poſſimus *relationis* ultimò intercedentis inter duas quantitates quam minimas, licet *abſoluta* magnitudo ipſarum quantitatum nec inveniri nec concipi poteſt. Res clariùs intelligetur in infinitè magnis : ſi quantitates duæ quarum data eſt differentia, augeantur in infinitum, dabitur harum ratio ultima, nempè ratio æqualitatis ; nec tamen idcircò dabuntur et ipſæ quantitates ultimæ, ſeu maximæ, quarum iſta eſt

ratio: hæ enim, cùm in immenſum excreverint, omnem calculum, imò omnem *abſolutæ* magnitudinis conceptum, longè tranſiliunt et prorsùs refugiunt.

33. Illuſtrari poſſunt hæc, paulò ſubtiliùs forsàn et abſtractiùs diſputata quam pro tyronum captu, ex conſiderationibus *Algebräicis* et *Geometricis*, et *Arithmeticis*.

1. In reductione æquationum *quadraticarum*, *Analyſ. Æquat.* § 30. prodiit duplex valor radicis, nempè $x = \frac{1}{2}p \pm \sqrt{\frac{1}{4}p^2 - q}$, eritque valor uterque realis, ubi $\frac{1}{4}p^2$ fit *major* quam q; uterque verò impoſſibilis, ubi fit *minor*; et ipſo in *limite*, ubi $\frac{1}{4}p^2$ fit *æqualis* q, erit unicus valor radicis; evaneſcente jam quantitate radicali $\sqrt{\frac{1}{4}p^2 - q}$.

2. In reſolutione *cubicarum* quoque, § 307, ipſo in *limite*, ubi $\frac{3}{4}p^2$ fit *æqualis* q, duplex valor factoris quadratici, $x = -\frac{1}{2}p \pm \sqrt{-\frac{3}{4}p^2 + q}$ in unicum paritèr coaleſcit.

34. Et ipſe *Landen*, in demonſtrationem ſuam *Theorematis Binomii*, purè Algebräicam (ut refert) clanculùm introducit methodum *limitum*, ſeu fluxionum, ubi ſingit quòd "*ſalvâ* æquatione *variari* poteſt y ad libitum, donec fiat $y = x$."—Analys. Æquat. p. 38.

Nec minùs *De La Grange*, naturam radicum æquationis optimè exponens, et theoriam æquationum Algebräicarum peritiſſimè promovens, ut docuimus Analys. Æquat. § 145—148, methodum *limitum* felicitèr ſanè introducit.

Hinc pendet omnis inventio *limitum* vel *radicum æquationis*, vel extremarum, vel intermediarum; quorum inventio inveſtigandis ipſis radicibus apprimè utilis eſt, per varias approximandi methodos.

35. In *ſectionibus conicis* quoque luculentiſſimè obtinet *methodus limitum*; inde enim pendet omnis theoria *circulorum* et *parabolarum* curvas alias *oſculantium*, ingenti cum commodo theoriæ motuum planetarum atque cometarum adhibita, à ſummis Analyſtis, *Newtono*, *Halley*, *Clairaut*, &c.

Nam, ſi concipiantur ſingulæ *ſectiones conicæ* gigni ex interſectionibus plani per axem coni perpetuò rotantis, cum ſuperficie conicâ vel ſuperficiebus conicis:

1. Sectio plani generantis, aut paralleli ad baſin coni, aut ſubcontrariè poſiti, erit *circulus*.

2. Circulus, rotante plano, ſtatìm migrabit in *ellipſin*; cujus eccentricitas, rotando, continuè augebitur, donec planum fiat parallelum lateri oppoſito coni ipſius; in hòc verò ſtatu mutationis, ellipſis viciſsìm, in *parabolam*, centro in infinitum abeunte, migrabit: deinde, rotante adhùc plano, parabola ſtatìm migrabit in *hyperbolam* atque *hyperbolas oppoſitas*, centro ſectionis jam extrinſecùs

tranſlato inter vertices; et hyperbolarum eccentricitas augebitur, donec, plano cum ipſo axi tandem coincidente, hyperbolæ oppoſitæ in *triangula verticalia* migrabunt. Atqui eadem contingent, ſed ordine contrario, donec, totâ rotatione peractâ, reſtituitur *circulus*.

3. Hinc conſtat *circulum* eſſe limitem naſcentium ellipſium; *parabolam*, evaneſcentium ellipſium et naſcentium hyperbolarum; et *triangulum*, evaneſcentium hyperbolarum. Sed calculus limitum longè facilior prodit quàm calculus ſectionum intermediarum; quoniam *circuli* omnes ſunt figuræ ſimiles; necnon *parabolæ omnes*; ſed ſingulæ ellipſium vel hyperbolarum ſpecies, rotando genitæ, ſunt inter ſe diſſimiles, et diverſos calculos, nempè, unaquæque ſpecies calculum ſibi proprium, requirunt.

36. Ex *rationibus arithmeticis*, hactenùs poſita quoque illuſtrare fas eſt.

1. *Si data quantitas* ($2a$) *dividatur inæqualitèr, exponente* (x) *partem intermediam variabilem; erit ratio facti ſub partibus inæqualibus* $(a + x) \times (a - x)$ *ad quadratum ſemiſſis, ratio minoris inæqualitatis*; propter defectum quadrati x^2. Per *Prop*. 5. *Lib*. 2. *Elem.*

Primò quidem, ratio *naſcentis* facti ſeu rectanguli, (quantitate quàm maximè inæqualitèr diviſâ) ad hoc quadratum, ferè infinita evadit, propter x ferè æqualem a; dein, *decreſcente* x, ratio rectanguli ad quadratum perpetuò accedet ad æqualitatem; et ultimò, *evaneſcente* x, ipſo in limite, fiet ratio æqualitatis.

Ex. gr. Sit $a = 101$, et primò, variabilis $x = 100$; erit rectangulum ad quadratum in ratione 201×1 ad 101×101; ſeu 201 ad 10201, ſeu 1 ad 50.7, hoc eſt, in ratione longè minore. Dein, ſit $x = 10$, et ratio rectanguli ad quadratum jam fiet 111×91 ad 101×101; ſeu 10101 ad 10201, ſeu 1 ad 1.009; propè ad æqualitatem accedente. Poſtremò ſit $x = 1$; evadétque ratio, 102×100 ad 101×101, ſeu 10200 ad 10201, ſeu 1 ad 1.000,09, proximè ad æqualitatem accedente.

37. "Res clariùs intelligetur in infinitè magnis."

Si eadem quantitas ($2a$) *perpetuò augeatur, deſignante jam* x, *augmentum variabile, erit ratio rectanguli* $(2a + x) \times x$ *ad quadratum* $(a + x)^2$, *ratio quoque minoris inæqualitatis*. Propter defectum quadrati conſtantis, a^2. Per *Prop*. 6. *Lib*. 2. *Elem.*

In hòc caſu quoque, *creſcente* x, ratio rectanguli ad quadratum, perpetuò ad æqualitatem accedit; et ultimò, *in infinitum aucto* x, fit ratio æqualitatis.

Ex. gr Sit $a = 1$; et primò ſit variabilis $x = 1$; erit rectangulum ad quadratum in ratione 3×1 ad 2×2; ſeu 3 ad 4, ſeu 1 ad 1.3333, &c. Dein, ſit $x = 10$; evadétque ratio 12×10 ad 11×11; ſeu 120 ad 121, ſeu 1 ad 1.008; propiùs ad æqualitatem accedens: denuò ſit $x = 100$; et fiet ratio 102×100 ad 101×101; ſeu 10200 ad 10201, ſeu 1 ad 1.000,08;

longè propiùs quàm in caſu ultimo. Jam verò ſit $x = 1000$; et ratio ejuſdem rectanguli ad dictum quadratum quam proximè ad rationem æqualitatis accedet; erit enim æqualis rationi 1002×1000 ad 1001×1001, ſeu 1002000 ad 1002001, ſeu 1 ad 1,0000008. Denique ſit valor x *infinitè magnus*; et fiet ultimò ratio rectanguli ad quadratum ratio æqualitatis; differentiâ conſtanti, a^2, $= 1$, tandem pro nihilo evadente reſpectu magnitudinum ipſarum adeò immenſarum: hæc igitùr differentia *quaſi* evaneſcit, licèt nunquam *actu* evaneſcit. Claram igitùr habere poſſimus ideam *relationis* inter duas quaſcunque magnitudines utcunque auctas vel diminutas in infinitum, licèt ipſarum magnitudinum *abſolutarum* ideam determinatam ſeu poſitivam nullatenùs effingere fas ſit, mortalibus, *finitas* tantùm *facultates* ſortientibus.

38. Quò meliùs intelligatur natura *rationum primarum et ultimarum*, quam hìc preſſiùs expoſui; principia *Newtoniana*, breviſſima et ſubtiliſſima quidem, quâ potui fidelitate evolvendo; placet inſupèr ſententias interpretum fuſiùs adducere; non ſolùm illuſtrandi gratiâ, verùm etiàm ex abundanti confirmandi hactenùs expoſita, atque obtrectatorum inſcitiam pleniſſimè redarguendi, ex peritiſſimis *Newtoni* diſcipulis.

I.—*Colſon's Account of the Method of Fluxions.*

39. "The *chief principle* upon which the Method of Fluxions is built is this very ſimple one, taken from the *Rational Mechanics*; which is, that *mathematical quantity*, particularly *extenſion*, may be conceived as generated by continued *local motion*; and that *all quantities* whatever, (at leaſt by *analogy* and *accommodation*,) may be conceived as generated after a like manner: conſequently, there muſt be *comparative* velocities [or rates] of increaſe and decreaſe during ſuch generations; whoſe *relations* are fixed and determinable, and may therefore (problematically) be propoſed to be found.

"This problem our author ſolves by the help of *another principle* not leſs evident; which ſuppoſes that quantity is *infinitely diviſible*, or that it may (*mentally* at leaſt) *ſo far* continually diminiſh, as at laſt, before it is *totally* extinguiſhed, to arrive at quantities which may be called *vaniſhing* quantities, or which are *infinitely little*, and leſs than any *aſſignable* quantity: or it ſuppoſes that we may form a notion, not indeed of *abſolute*, but of *relative* and *comparative infinity*.

"It is a very juſt exception to the Method of *Indiviſibles* [of *Cavalerius*], as alſo to the foreign *Infiniteſimal* Method [of *Leibnitz*], that they have recourſe *at*

once to infinitely little quantities, and infinite orders and gradations of theſe, not relatively but abſolutely ſuch: they aſſume theſe quantities *ſimùl et ſemèl*, without any ceremony, as quantities that *actually* and *obviouſly* exiſt, and make computations with them accordingly; the reſult of which muſt needs be as *precarious* as the abſolute exiſtence of the quantities they aſſume.

“ *Abſolute* infinity, as ſuch, can hardly be the object either of our conceptions or calculations; but *relative* infinity may, under a proper regulation. Our author obſerves this diſtinction very ſtrictly, and introduces none but *infinitely little* quantities that are *relatively* ſo: which he arrives at, by beginning with finite quantities, and proceeding by a gradual and neceſſary progreſs of diminution: his computations always commence by *finite* and *intelligible* quantities; and then, at laſt, he inquires what will be the reſult, in *certain circumſtances*, when ſuch or ſuch quantities are diminiſhed *in infinitum*. This is a conſtant practice even in *Common Algebra* and *Geometry*; and is no more than deſcending from a *general propoſition* to a *particular caſe* which is certainly included in it.”

40. “ And from theſe *eaſy* principles, managed with a vaſt deal of ſkill and ſagacity, he deduces his *Method of Fluxions*: which if we conſider only *ſo far* as he himſelf has carried it, together with the *application* he has made of it, either here or elſewhere, directly or indirectly, expreſsly or tacitly, to the moſt curious diſcoveries in *art* and *nature*, and to the ſublimeſt *theories*, we may deſervedly eſteem it as the *greateſt work of genius*, and as the nobleſt effort that ever was made by the human mind.”

41. “ Indeed it muſt be owned, that many uſeful *improvements* and *new applications* have been *ſince* made by others, and probably will be *ſtill* made, every day: for it is no mean excellence of this method, that it is doubtleſs ſtill *capable* of a greater degree of perfection, and will always afford an inexhauſtible fund of curious matter, to reward the pains of the ingenious and induſtrious *analyſt*.”

42. “ It ſhould be well conſidered by theſe *gentlemen objectors*, that the great number of *examples* they will find here, to which the Method of Fluxions is ſucceſsfully applied, are ſo many *vouchers* for the truth of the principles on which that method is founded: for the deductions are *always conformable* to what has been derived from other *uncontroverted* principles; and therefore muſt be acknowledged as *true*.

“ This argument ſhould have its due weight even with ſuch as *cannot*, as well as with ſuch as *will not* enter into the proof of the principles themſelves. And the *hypotheſis* that has been advanced to evade this concluſion—*of one error in reaſoning being ſtill corrected by another equal and contrary thereto*—and that, *ſo regularly*, *conſtantly*, *and frequently*, as it muſt be ſuppoſed to do here;—this hypotheſis, I ſay, ought not to be ſeriouſly refuted, becauſe I can hardly think it is ſeriouſly propoſed.”——*Preface to Colſon's Method of Fluxions*, p. 11.

II.—*Mac-*

II.—*Maclaurin's Account of the Method of Fluxions.*

43. "We would not be underſtood to affirm that the Methods of *Indiviſibles* and *Infiniteſimals*, by which *ſo many unconteſted truths* have been diſcovered, are without a foundation: we acknowledge further, that there is ſomething *marvellous* in the doctrine of *Infinites*, that is apt to pleaſe and tranſport us; and that the Method of *Infiniteſimals* has been proſecuted *of late*, with an acuteneſs and ſubtlety not to be paralleled in any other ſcience. But **Geometry** is beſt eſtabliſhed on *clear* and *plain principles*; and theſe ſpeculations are ever obnoxious to ſome difficulties: If the greateſt accuracy has been always required in this ſcience, in reaſoning concerning *finite* quantities, we apprehend that Geometricians cannot be *too ſcrupulous* in admitting or treating of *infinites*, of which our ideas are *ſo imperfect*.

44. "They who have made uſe of *infinites* and *infiniteſimals* with the greateſt liberty, have not agreed as to the *truth* and *reality* they would aſcribe to them.—*Cavalerius*, the ingenious author of the Method of *Indiviſibles*, diſcovered a method which he found to be of very extenſive uſe, and of an eaſy application for meaſuring and comparing planes and ſolids, and would not deprive the world of ſo valuable an invention. In propoſing it, he ſtrove to avoid the ſuppoſing magnitude to conſiſt of *indiviſible* parts, and to abſtract from the conſideration of *infinity*:—"Quoad continui compoſitionem, manifeſtum eſt ex præoſtenſis, ad ipſum *ex indiviſibilibus* componendum, nos minimè cogi: ſolùm enim *continuò ſequi indiviſibilium proportionem*, et *è converſo*; intentum fuit: quòd quidem cùm utràque poſitione ſtare poteſt." Tandem verò dicta *indiviſibilium aggregata* non ita pertractavimus, ut *infinitatis rationem* propter infinitas lineas ſeu plana ſubire videantur, &c."—*Cavalerii Geom. Indiviſ. Lib.* 7. *Præf.*——But he acknowledged that there remained *ſome difficulties* in this matter, which he was not able to ſolve:—and he ſpeaks as if he foreſaw that it ſhould be afterwards delivered in an unexceptionable form, which might ſatisfy the moſt ſcrupulous Geometrician; and leaves this *gordian knot* (as he expreſſes himſelf) to ſome *Alexander*."

45. "The celebrated Mr. *Leibnitz*, juſtly eſteemed for his various writings, owns *infinites* to be no more than *fictions*:—"On s'embaraſſe dans les ſéries des nombres qui vont à *l'infini*: On conçoit un *dernier terme*, un *nombre infini*, où *infiniment petit*: mais tout celà ne ſont que *des fictions*. Tout nombre eſt *fini* et *aſſignable*; toute ligne l'eſt le même."—*Eſſai de Théodicée*, diſc. prélim. § 70.—They who treated of infinites *before him* proceeded, as he obſerves, with a *timorouſneſs* which the contemplation of ſuch an object naturally inſpires:—

"Quand on y étoit arrivé, on s'arreſtoit avec un eſpèce *d'effroy* et de *ſainte horreur*—on regardoit *l'infini* comme un *myſtère*, qu'il falloit *réſpecter*, et qu'il *n'étoit pas permis d'approfondir.*"—"*They* ſtopt when they came to *infinity* with a ſort of *holy dread*, and reſpected it as an *unfathomable myſtery*."—*He* adventures farther, in order to diſcover the ſource, and penetrate into the firſt principles of GEOMETRICAL truth. INFINITY, according to him, *is the great trunk from which its various branches are derived, and to which they all lead.* In this great purſuit, he diſplays *infinite* and *finite*, with a *freedom* that puts us in mind of the ancient poet [*Homer*] and his gods—whom he repreſents with *the paſſions of men*, and mingles in their *battles*."

"We doubt not that if a full and perfect account of all that is *moſt profound* in the HIGH GEOMETRY *could* have been deduced from the *Doctrine of Infinites*, it might have been expected from this author: but our *ideas of infinites* are too *obſcure* and *inadequate* to anſwer this end: and there are many things advanced by *all thoſe* who have applied them with great freedom in GEOMETRY, that give ground to a remark *like* to M. de *St. Evremond*'s, when he obſerves, that "it is ſurpriſing to find the *ancient poets* ſo ſcrupulous to preſerve *probability* in actions purely *human*; and ſo ready to violate it, in repreſenting the actions of *the Gods*." —Some have not only admitted *infinites* and *infiniteſimals* of *infinite* orders; but have diſtinguiſhed even *nothings* into various kinds!—and if ſuch liberties continue, it is not eaſy to foreſee what *abſurdities* may be advanced as *diſcoveries* in what is called the SUBLIME GEOMETRY."—"PHILOSOPHY probably will always have it's *myſteries*; but theſe are to be avoided in GEOMETRY, [as *far as is poſſible, and as much as in us lieth*]; and we ought to guard againſt abating from it's ſtrictneſs and evidence the rather, that an ABSURD PHILOSOPHY *is the natural product of a* VITIATED GEOMETRY."

46. "Sir ISAAC NEWTON accompliſhed what CAVALERIUS had wiſhed for, [and avoided the rock on which the daring *Leibnitz* had ſplit] by inventing the *Method of Fluxions*; and propoſing it in a way that admits of *ſtrict demonſtration*: which requires the ſuppoſition of no quantities but ſuch as are *finite*, and eaſily conceived. The computations in this method are the ſame as in the Method of *Infiniteſimals*; but it is founded on accurate principles agreeable to the ANCIENT GEOMETRY: In it, the *premiſſes* and *concluſions* are equally accurate; no quantities are rejected as *infinitely ſmall* *, and no part of a *curve* is ſuppoſed to *coincide with a right line*. The excellency of this method has not been ſo *fully deſcribed*, or ſo *generally attended to*, as it ſeems to deſerve; and it has been ſometimes repreſented as on a level, in all theſe reſpects, with the Method of *Infiniteſimals*. The chief deſign of this treatiſe, is to ſhew it's advantages in a clearer and fuller light, and to promote the deſign of the great inventor, by

---

* Compare *Newton*'s Demonſtration of the *Fluxion of a Rectangle*, § 84, 85, with *Euler*'s, derived from the *Infiniteſimal Method*, § 87, to which it is infinitely ſuperior in accuracy and elegance.

eſtabliſhing the HIGHER GEOMETRY on plain principles, perfectly conſiſtent with each other, and with thoſe of the *ancient Geometricians*."—*Maclaurin's Introduction to Fluxions*.

De Applicatione Methodi Rationum Primarum et Ultimarum ſeu Limitum, ad Quadraturam Curvarum, atque ad Mechanicam Rationalem.

---

47. HUJUS applicationis elegantia quædam proferuntur exempla à *Newtono* in *Lemmatibus inſequentibus*: LEM. II. et III. *Si polygonorum rectilineorum figuræ cuivis curvilineæ ſimiliter inſcriptorum et circumſcriptorum augeatur numerus laterum correſpondentium, et diminuatur magnitudo in infinitum, ità ut differentia polygoni inſcripti et circumſcripti tandem fiat minor quâvis datâ: ultimæ rationes quas habent polygonum inſcriptum et circumſcriptum, ad ſe invicem, (et multò magis ad figuram curvilineam intermediam) erunt rationes æqualitatis.*

Immediatè et nullo negotio ſequitur hoc, ex *Lemmate primo* fundamentali; ídque ſive polygonorum latera ſint *æqualia*, ut in *Lemmate ſecundo*; ſive *inæqualia*, ut in *Lemmate tertio*.

Cor. 1. Hinc *polygona inſcripta et circumſcripta*, tandem evaneſcente differentiâ, *ultimò coincident omni ex parte, cum figurâ curvilineâ*.

Cor. 2. Et propterea, *polygona ultima*, quoad perimetros ſuos, non ſunt *rectilinea*, ſed rectilineorum *limites curvilinei*.

48. Atqui hæc eſt ipſa *Methodus Exhauſtionum* quam exercebant *Geometræ veteres*, in compendium redacta, et longè promota, ab hoc artifice ſummo; ſicut luculentiùs oſtenditur in *Introductione Maclaurinianâ*:

"The *Method of Exhauſtions* was that by which the Ancients demonſtrated all their theorems for meaſuring and comparing *curvilinear* figures. It is often ſaid that *curve lines* have been conſidered by them as *polygons of an infinite number of ſides*; but this principle no-where appears in their writings: we never find *them* reſolving any figure or ſolid into *infinitely ſmall elements*: on the contrary, they ſeemed to avoid ſuch ſuppoſitions; as if they judged them *unfit* to be received into GEOMETRY, when it was obvious that their demonſtrations might have been ſometimes *abridged* by admitting them."

"They conſidered *curvilinear areas* as the *limits* of circumſcribed or inſcribed *figures* of a *more ſimple kind*, which *approach* to theſe limits (by a biſection of

lines or angles that is continued at pleaſure); *ſo that the difference betwixt them may become leſs than any given quantity*: The inſcribed or circumſcribed figures were always conceived to be of a number and magnitude that is *aſſignable*; and from what had been ſhewn of *theſe figures*, they demonſtrated the menſuration or the proportions of the *curvilinear limits themſelves*, by arguments *ab abſurdo*. They had made frequent uſe of demonſtrations of this kind from the beginning of the ELEMENTS [as where the affections of *ſimilar polygons*, found in the firſt ſix books, are tranſferred to *circles* in the twelfth book.] And theſe are in a particular manner adapted for making a tranſition from *right-lined* figures to ſuch as are bounded by *curve lines*. By admitting *them* only, they eſtabliſhed the more difficult and ſublime parts of their *Geometry*, on the ſame foundation as the firſt elements of the ſcience. *Nor could they have propoſed to themſelves a more perfect model*."

49. "We have already obſerved how ſolicitous *Archimedes* appears to be, that his demonſtrations ſhould be found to depend on thoſe principles only that had been *univerſally received* before his time. In his Treatiſe of the *Quadrature of the Parabola*, he treats of a progreſſion whoſe terms decreaſe conſtantly in the proportion of 4 to 1 (which we expreſſed by the trapezia CD*ba*, DG*cb*, GR*dc*); but he does not ſuppoſe this progreſſion to be continued to infinity, or mention the ſum of an infinite number of terms; though it is manifeſt, that all which *can* be underſtood by thoſe who aſſign that ſum, was *fully* known to him. He contented himſelf with demonſtrating this plain property of ſuch a progreſſion: That *the ſums of the terms continued at pleaſure, added to the third part of the laſt term, amount always to four-thirds of the firſt term*: (as, for inſtance, in the *Quadrature* of the *Parabola*, *Introduct*. p. 27—29. Pl. 3. *Fig*. 13. the ſum of the trapezia CD*ba*, DG*cb*, GR*dc*, added to one-third part of the laſt trapezium GR*dc*, amounts always to four-thirds of the firſt trapezium CR*da*, or to it's limit, the triangle CA*a*;) but while this firſt trapezium approaches continually to it's limit, the inſcribed polygon at the ſame time is approaching to the *parabolic* ſegment it's limit; and theſe are equal to each other, as there demonſtrated, or in *Hamilton's Conic Sections*, (*De Quadraturâ Parabolæ*, according to *Archimedes*.)—After all the methods that have been propoſed for demonſtrating the Quadrature of the Parabola, this *of the inventor himſelf ſeems to have a particular accuracy and elegance*: he does not ſuppoſe the chords of the curve to be biſected to infinity; ſo that, after an infinite biſection, the *inſcribed polygon* might be ſaid to *coincide* with the *parabola*. Theſe ſuppoſitions had been new to the geometricians in his time; and ſuch he appears to have avoided carefully.—On this occaſion, he ſhews *how to find* the ſum of any *number of terms that decreaſe conſtantly in the proportion of four to one*: and by this example of a *geometrical progreſſion*, (as it is commonly called,) opened up a ſubject, which has been treated at great length by the modern geometricians."

50. "But even *Archimedes* himſelf has not eſcaped the cenſures of ſome writers, *Hobbes*, *Joſeph Scaliger*, &c. who, being *unſkilful* in Geometry, and *unable to reconcile their own conceits with his demonſtrations*, have repreſented him as in an error, and "miſleading mathematicians by his *authority*."—*Plutarch*, however, celebrates the *ſimplicity* and *plainneſs* with which he treats the moſt difficult and abſtruſe queſtions :

Ου γαρ εϛιν εν Γεομετρια χαλεπωτερας και βαρυτερας ὑποθεσεις εν ἁπλȣϛεροις λαβειν και καθαρωτεροις ϛοιχειοις γραφομενας.

"*For it is impoſſible to find more difficult and weighty ſubjects written in ſimpler and purer elements.*"

"He appears indeed to have been more fond of preſerving to the ſcience all it's *accuracy* and *evidence*, than of advancing *paradoxes*."

"And although the purſuit of *general* and *eaſy* methods may have induced ſome moderns to make uſe of *exceptionable* principles, and the vaſt extent which the ſcience has of late acquired, may have occaſioned their propoſing incompleat demonſtrations, yet the *firſt eſſays* were deduced from a careful attention to his ſteps :—"*C'etoit en obſervant de près la marche d'* ARCHIMEDE, *qu'il* (M. De Roberval) *étoit arrivé à cette ſublime et merveilleuſe ſcience.*"—*Acad. Scient.* an. 1693.—This is generally acknowledged by the writers of that time."

51. "Hence it was with caution that the Doctrine of *Indiviſibles* was at firſt employed in Geometry by *Cavalerius* ; therefore he ſubjoined more *unexceptionable* demonſtrations to thoſe he had deduced from his own principles : and the diſputes which enſued (the firſt of any moment that were known between *geometricians*) juſtified his precautions."

52. "Geometricians of the firſt rank had recourſe to this method of *limits* long ago, on ſeveral occaſions, as a method of the ſtricteſt kind : M. de *Fermat*, in a letter to *Gaſſendi*, and M. *Huygens*, in his *Horologium Oſcillatorium*, have employed it for compleating the demonſtrations of ſome theorems that were propoſed by *Galileo*, and proved by him in a leſs accurate manner ; and Dr. *Barrow* has demonſtrated by it a theorem concerning the *Tangents of Curve Lines*."

53. In *Quarto Lemmate* demonſtrat Newtonus *rationes limitum curvilineorum, ultimò eſſe eaſdem ac polygonorum inſcriptorum*.

Nam ut *partes ſingulæ* polygonorum inſcriptorum *ad ſingulas*, ita (componendo) fit *ſumma omnium* ad *ſummam omnium* ; et proindè (per *Lem.* 3. *Cor.* 2. præced.) *limes curvilineus ad limitem curvilineum*. Q. E. D.

Cor. Hinc, ſi *duæ cujuſcunque generis quantitates*, in *eundem partium numerum* utcunque dividantur ; et partes illæ, ubi numerus earum augetur et magnitudo

diminuitur in infinitum *datam obtineant ad invicem rationem*, prima ad primam, ſecunda ad ſecundam, cæteræque ſuo ordine ad cæteras: erunt *tota* ad invicem *in eâdem illâ datâ ratione.* Nam, ſi ſumantur tota inter ſe ut partes, erit totum prius ad ſummam partium, et totum poſterius ad ſummam partium, in ratione æqualitatis; et proindè, per hypotheſin, in ultima ratione partis ad partem.

Q. E. D.

54. Ope hujus *principii generaliſſimi*, tàm GEOMETRIAM quàm ALGEBRAM permeantis, emendare licet demonſtrationem *Tacquetianam*, *Prop.* 1. *Lib.* 6. *El.* Nempè "*triangula æquialta*, &c. *eſſe ad invicem in ultimâ ratione baſium*;" quæ deficit, ubi de *incommenſurabilibus* agitur; cum "*indicium proportionis*" pro *Euclidiano* malè ſubſtitutum à *Tacquet*, *commenſurabilibus* tantùm reſtringitur.

Sumptis partibus ſimilibus DEG et DG trianguli prioris DEF, et baſis prioris DF; ſi ſubſtituantur hæ partes in triangulum poſterius ABC et baſin poſteriorem AC reſpectivè, quoties fieri poteſt; ubi numerus partium augetur et magnitudo diminuitur in infinitum, ultimò *datam obtinebunt ad invicem rationem*: et proindè, erunt *tota* ad invicem in *eâdem illâ datâ ratione*: hoc eſt, erit triangulum prius DEF ad poſterius ABC, ut baſis prior DF ad poſteriorem AC. Q. E. D.

55. Quàm iniquæ igitùr ſunt in *Newtonum* criminationes, quaſi "*methodum recentem*," "veteribus *ignotam*," "*alienis* principiis fundatam, à doctrinâ *motûs* tranſlatis," iiſdémque "*minimè neceſſariis*" excogitaſſet!—Pudet equidèm tales cavillationes unquam admiſiſſe analyſtas ex peritiſſimis *Landen*, *D'Alembert*, et *Cenſorem* illum *Profeſſorium*, præ incuriâ et oſcitantiâ ſaltèm, ſi non ex invidiâ oriundas. Maximè autem pudet viri clariſſimi *De La Grange*:—nonne *dormitat bonus*, ubi profitetur "ſe traditurum *functionum analyticarum* theoriam, ab omni conſideratione *limitum* immunem!"—(ſi fides habenda ſit iſti *Cenſori* apud THE MONTHLY REVIEW.)

Dic quibus in terris——
——*et eris mihi magnus Apollo!*

56. Atqui luce clariùs conſtat, ex his documentis, principia infirma et vacillantia *methodi differentialis*, quæ primò poſuerat *Leibnitz*, ex notionibus ſuis vagis et incongruis de *infiniteſimis*, (ſeu quantitatibus infinitè parvis) gradatìm et lentè purgâſſe, et tacitè, diſcipulos ſuos; atque ad *Newtonianam veritatem*, atque *rigorem Geometricum*, tandem et quaſi proprio jure, perfeciſſe: nunc verò reclamare audent ingrati, debito ſuo honore *Newtonum*, ſucceſſorem dignum veterum analyſtarum, *Archimedis*, *Apollonii* et *Euclidis*, iniquiſſimè defraudantes!

57. Demonſtrârat *Euclides*, *Prop.* 20. *Lib.* 6. *El.* quod "*Polygonorum ſimilium latera omnia quæ ſibi mutuò reſpondent ſunt proportionalia*, et, 2. *Areæ ſunt in duplicatâ ratione laterum*:" hanc proprietatem inſignem ad *circulum* tranſtulit

Archimedes; et ipſe Euclides in *Prop.* 2. *Lib.* 12. *El.* Et generalitèr, extendit *Newtonus* ad *curvas quaſcunque*, in *Lemmate quinto*; eodem ratiocinio poſito, idque, ſive *latera* ſint *curvilinea*, ſive *rectilinea*.

58. In *Lemmate ſeptimo*, demonſtrat *Newtonus*, (per evaneſcentiam anguli contactus, *Lem.* 6.) quod *ultima ratio arcûs quam minimi*, *chordæ* et *tangentis eſt ratio æqualitatis*; paritérque *ſemiarcûs*, *ordinatæ* et *abſciſſæ tangentis*; vel *chordæ ſemiarcûs*, *ordinatæ*, &c. Et propterea *hæ omnes lineæ* in omni de *rationibus ultimis* argumentatione pro ſe invicèm uſurpari poſſunt.

59. In *Lemmate octavo*, quod *ultima forma triangulorum evaneſcentium*, (ubi ex rectis à centro quodam quaſi vertice, ad baſin productis, ſive hæc baſis, ſit vel arcus, vel chorda vel tangens, fiunt hæc triangula) *eſt ſimilitudinis*, et *ultima ratio æqualitatis*. Et hinc *triangula illa* in omni de *rationibus ultimis* argumentatione pro ſe invicèm uſurpari poſſunt.

60. In *Lemmate nono*, demonſtrat (per *Lem. quintum*), *quod areæ triangulorum curvilineorum ſimilium* ex curvarum tangentibus ſeu ſecantibus, et *ordinatis* ſeu ſubtenſis externis angulorum contactûs factorum, *erunt ultimò ad invicèm in duplicatâ ratione laterum*.

61. Elegantiſſimâ tranſlatione factâ ad *Mechanicam Rationalem*, in *Lemmate decimo* demonſtrat, quòd *ſi corpus*, *urgente vi quâcunque finitâ*, *moveatur*; ſive, 1. *vis illa determinata et immutabilis ſit*; ſive, 2. *eadem continuò augeatur vel continuò diminuatur*; *erunt ſpatia ipſo motûs initio in duplicatâ ratione temporum*.

Nam, ſi *tempora* motûs exponantur per *abſciſſas*, et *velocitates genitæ* per *ordinatas externas*, (ſeu ſubtenſas angulorum contactûs) ſpatia, hâc vi urgente, deſcripta ritè exponentur per *areas triangulares* curvilineas híſce ordinatis deſcriptas; et proinde (per *Lem.* 5 et 9.), erunt in duplicatâ ratione laterum correſpondentium, h. e. *abſciſſarum* ſeu *temporum*.

Cor. 1. Et quoniam virium diverſarum intenſitates ritè exponuntur per *velocitates* quas generant, erunt *hæ vires*, ut *ſpatia*, ipſo motûs initio deſcripta directè et *quadrata temporum* inversè; per *Prop.* 23. *Lib.* 6. *El.*

62. In *Lemmate undecimo* demonſtrat *Newtonus*, quòd *ordinata externa* (ſeu *ſubtenſa evaneſcens anguli contactûs*) *in curvis omnibus curvaturam finitam ad punctum contactûs habentibus*, *eſt ultimò in ratione duplicatâ chordæ arcûs contermini*.

Nam, ex naturâ *circulorum æquicurvorum*, circà has areas triangulares deſcriptorum, erunt *ordinatæ externæ*, ſeu *circulorum ſagittæ*, his proportionales, ut quadrata *chordarum* arcuum conterminorum directè, et *diametri* circulorum inversè; ſed, ultima ratio diametrorum, evaneſcente differentiâ, ſit ratio æqualitatis; adeóque ordinatæ externæ erunt ut quadrata chordarum.

Cor. 1. Hinc, *ſagittæ curvarum* quoque, quæ *chordas arcuum duplorum* biſecant, et ad datum centrum motûs convergunt, *ſunt etiàm ultimò ut quadrata arcuum conterminorum.* Nam *ſagittæ illæ* ſunt ut *ordinatæ externæ*, et *arcus* cùm *chordis* ultimò coincident, per *Lem.* 7.

Cor. 2. Ideóque *ſagittæ illæ ſunt in duplicatâ ratione temporum quibus corpus datâ velocitate deſcribit arcus quam minimos.* Nam datâ velocitate, hi arcus ſunt ut tempora; et proindè quadrata arcuum ut quadrata temporum.

63. Atque ſunt hæc *principia mathematica generaliſſima*, quorum ope demonſtrat *Newtonus* in ſequentibus propoſitionibus, *Sect.* 11. *rationes virium centripetarum* ad invicèm; *legéſque* ſecundùm quas corpora circà commune centrum motûs revolventia, accelerantur aut retardantur pro diverſis à centro diſtantiis. Vide § 88, 89 et 103.

De Fluxionum Elementis Primis.

64. Quo magìs pateat hujúſce Methodi, ſive *Fluxionalis* ſive *Differentialis*, evidentia et certitudo, viſum eſt "*fundamentum* totius methodi" inſpicere et perſcrutari, quale illud ſtruxit artifex ille egregius in *Lemmate ſecundo Lib.* 2. *Princip.* ulteriúſque obitèr expoſuit, in *Analyſi per Series Numero Terminorum Infinitas*, atque in *Quadraturâ Curvarum*;—quæ tria opera "*chordam triplicem haud facilè diſrumpendam*" conſtituunt, vel viribus obtrectatorum *Britannorum* vel *Gallorum*, vel aliorum quorumvis contrà hanc doctrinam infauſtè combinatorum.

65. Terminos fluxionum *technicos* ſcitè definit *Newtonus*, et expoſitione ſatìs dilucidâ diſtinguit:

1. "*Genitam* voco, quantitatem omnem quæ ex lateribus vel terminis quibuſcunque, in ARITHMETICA per multiplicationem, diviſionem et extractionem radicum; in GEOMETRIA, per inventionem vel contentorum et laterum, vel extremarum et mediarum proportionalium, ſine additione et ſubductione generantur: ejuſmodi quantitates ſunt *facti*, *quoti*, *radices*; *rectangula*, *quadrata*, *cubi*, *latera quadrata*, *latera cubica*, et ſimiles. Has quantitates, *ut indeterminatas* et inſtabiles, et *quaſi motu fluxúve perpetuo* creſcentes vel decreſcentes, hìc conſidero: et

2. "Earum incrementa vel decrementa momentanea, ſub nomine *momentorum* intelligo; ita ut *Incrementa*, pro *momentis addititiis* ſeu *affirmativis*; ac *Decrementa*, pro *ſubductitiis* ſeu *negativis*, habeantur.

66. "Cave tamen intellexeris [ſub nomine *momentorum*] *particulas finitas:* particulæ finitæ non ſunt *momenta*, ſed *quantitates ipſæ* ex momentis genitæ; [per vocem *momenta*] intelligenda ſunt *principia jamjam naſcentia* finitarum magnitudinum. Neque enim ſpectatur in hoc lemmate *magnitudo* momentorum, ſed *prima naſcentium proportio*."

Idem quoque fuſiùs exponit in *Analyſi per Æquationes*; doctiſſimo *Colſon* ita Anglicè eam reddente:

"The *moments* of *flowing* quantities are their *indefinitely ſmall* parts, by the acceſſions of which, in *indefinitely ſmall portions of time*, they are continually increaſed."

3. "Now theſe quantities, which I conſider as gradually and indefinitely increaſing, I ſhall hereafter call *Fluents*, or flowing quantities."

67. 4. "Whereas only quantities of *the ſame kind* can be compared together, and alſo their *velocities* [or *rates*] of increaſe or decreaſe; therefore, in what follows, I ſhall have no regard to *time*, formally conſidered; but I ſhall ſuppoſe *ſome one* of the quantities propoſed (being of the ſame kind) to be increaſed by an *equable* quantity, to which *the reſt* may be referred, *as it were*, to *time:* and therefore, by way of *analogy*, it may not improperly receive the name of *Time*. When, therefore, the word *time* occurs in what follows, (which, for the ſake of *perſpicuity* and *diſtinction*, I have ſometimes uſed,) by that word, I would not have it underſtood as if I meant time in its *formal* acceptation, but only that *other quantity*, by the equable increaſe or flux whereof time is *expounded* and *meaſured*."

68. 5. "Eodem recidit, ſi loco *momentorum* [ipſorum,] uſurpentur vel *velocitates* incrementorum ac decrementorum, (quas etiàm, "*motûs mutationes*," et "*fluxiones* quantitatum," nominare licet,) vel *finitæ quævis quantitates*, velocitatibus hiſce *proportionales*."

Et paritèr quoque, ſcribit *Newtonus* in *Quadraturâ Curvarum:*—"*Fluxiones* ſunt quàm proximè ut fluentium *augmenta æqualibus temporibus quàm minimis genita*; et, ut accuratè loquar, ſunt *in primâ ratione momentorum naſcentium*."

And again, in the *Analyſis:*—"The *moments* of flowing quantities—are *as* the *velocities* [or *rates*] of their flowing or increaſing;"—"and the *velocities* [or *rates*] by which every fluent is increaſed, by it's generating motion [whatever that may be], I may call *Fluxions*; or ſimply, *Velocities* or *Celerities*."

69. Ex his documentis undique congeſtis et intertextis, luculentiſſimè conſtat, quantâ peritiâ, ſolicitudine et cautione, methodi ſuæ generalis fundamentum poſuit Newtonus, in *definitionibus*, *diſtinctionibúſque* accuratiſſimis; "*perſpicuitatis*" (ut ipſe dicit) quàm maximè ſtudioſus, in re longè difficillimâ et ſubtiliſſimâ.

liſſimâ. Abundè autem liquet, eum uſurpâſſe vocem “ *velocities*,” (quæ tantas tragœdias excitavit), in latiore ſenſu, pro *rationibus* creſcendi vel decreſcendi—“ *rates* of increaſe or decreaſe.”—Et inde ſenſu generaliſſimo definiri poſſunt *fluxiones—Menſuræ quævis rationum ſecundùm quas quantitates fluentes quælibet ejuſdem generis, quolibet in ſtatu mutationis, utcunque variantur.*

70. Quidni autem eadem licentia *Newtono* concedatur, quam ſibi ſumebant *Analyſtæ veteres?* Cur non, æquo ſaltèm jure, *Newtono* fas fuerit, in rebus geometricis principium geometricum adhibere, et “*genitas*” quaſcunque quantitates, ex fluxu, ſeu “ *quaſi motu perpetuo*,” oriundas, conſiderare; quàm *Archimedi*, *Apollonio*, et *Euclidi*, concipiendi *lineam* quaſi ex motu *puncti*, *ſuperficiem*, quaſi ex motu parallelo *lineæ*, et *ſolidum*, quaſi ex motu parallelo *ſuperficiei*, reſpectivè genita? ídque, *illuſtrandi* gratiâ, nimirùm, in rebus *metaphyſicis* à ſenſibilibus valdè abſtractis? Nonne GEOMETRIA ipſa, exiguos fines *definitionis nominis* aſpernans, longè, imò longiſſimè, ultrà “ *menſurationem terræ*, *ſive agrorum*,”—“ *land-ſurveying* ”—in remotiſſimas cœlorum regiones, ultrà orbem *planetarum*, *cometarum* atque *aſtrorum*, ad ipſam *viam lacteam* excurrit—fræni impatiens, “ nec audit habenas?” Et quidnì vocabulum “ *fluxio*,” quoque, in ſenſu latiſſimo atque ampliſſimo, ad omnigenas *functiones*, MECHANICÆ RATIONALIS,—*vires*, *pondera*, *denſitates*, *reſiſtentias*, *reflexiones*, *refractiones*, *infractiones*, *aberrationes*, &c. ſeu quaſcunque quantitatum mutationes, ſecundùm magnitudines et directiones, ritè exponendas, extendatur et amplietur?—De *definitionibus nominum* diſputare, (non *κατα φυσιν* ſeu “ *ſecundùm rei ipſius naturam*,” ſed *κατα ϑεσιν*, “ *pro definientis arbitrio* ”—vel *inſtituto*)—puſilli eſt ingenii, *minutíque philoſophi.*—Eodem redeunt *Fluxiones* NEWTONI et *Functiones* DE LA GRANGE; ut ex definitione vocis poſterioris, apud THE MONTHLY REVIEW, conſtabit:

71. “ A *function* of one or more quantities, is *every expreſſion* in which theſe quantities enter *after any manner whatever* (combined, or not, with other quantities of given or variable values), *while* the quantities of the *function* may receive *all poſſible values*.”

72. Eodem quoque recidit *Methodus Differentialis*, (definiente analyſtâ periтiſſimo, *Euler*, in ſuis *Inſtitutionibus*) ac ipsa *Fluxionalis*:

“ *Methodus determinandi rationem incrementorum evaneſcentium, quæ functiones quæcunque accipiunt*; DUM *quantitati variabili cujus ſunt functiones, incrementum evaneſcens tribuitur*.”

Velùt ſi quantitatis variabilis x, incrementum ſit i; ubì hæc quantitas ſit $x + i$, fiet hujus quadratum, $x^2 + 2xi + i^2$. Erit igitùr

$$\textit{Increm. } x : \textit{increm. } x^2 :: i : 2xi + i^2;$$

vel, dividendo per i, —— :: 1 : $2x + i$.

Et proindè ratio 1 ad $2x$, exponet rationem incrementi lateris variabilis x ad incrementum quadrati variabilis x^2, ipſo in *limite*, ubi i evaneſcit, accuratè; antequàm evaneſcit, quàm proximè tantùm. At *limes rationis horum incrementorum evaneſcentium* ſolummodò in *Calculo Differentiali*, *Eulero* judice, ſpectatur.

73. Quis non videt analyſta benè cordatus, ad amuſſim quadrare Methodum *Fluxionalem* cum *Methodo Differentiali*, tàm *Euleri* quàm *De La Grange?*—et methodo utrique æquè competere conſiderationem *temporis* inter creſcendum decreſcendúmve elapſi?—ſi non "*formalitèr*" quidem, ſaltèm per *analogiam*, ſive per *accommodationem*, et illuſtrandi gratiâ?—et quale aut quantum intercedat diſcrimen (intelligo verum diſcrimen, non nomini tenùs) inter "*momenta*" Newtoni, et "*incrementa evaneſcentia*" Euleri, detegat, oro, analyſtarum vel perſpicaciſſimorum curioſa felicitas, in ſæcula ſæculorum.

74. Non idcircò niſi imperitè objicientis eſt "*animadverſio* docti *Torelli*," quam commendat M. R. quaſi—*inter exempla admodùm perſpicuæ validaéque argumentationis*" annumerandam:

"*Neque verò quidquam agunt, qui*, infiniteſimas quantitates *rejicientes*, momenta quædam *earum loco* GEOMETRIÆ *inferunt*; *eâque ita vocant, quòd concipiantur* momento temporis *finitæ magnitudinis fluxu oriri: Si enim* momenta hæc *vires ſuas fluendo explicant, non aliud ſunt quàm* ipſæ *quas rejiciunt* infiniteſimæ quantitates; *ſin autèm nituntur neque tamen ultrà tendunt*, novum vocabulum *fruſtrà inducitur, dum id quod* nihil *eſt* momentum *appellatur.*"

At ruit objectio ſuis viribus everſa. Nam, 1. *momentum* non eſt *nihil*, ut ex definitione conſtat; nec, 2. eſt *infiniteſima quantitas*, cum *magnitudinis finitæ*, ſeu determinatæ, (licet *quàm minimæ*) eſſe concipiatur; nam, ut accuratiſſimè loquitur *Newtonus*, eſt "*augmentum fluentis tempore quàm minimo genitum*;" neque, 3. ſi valida ſit, minùs hoſtilitèr in Methodum *Differentialem*, quàm *Fluxionalem*, invehitur, iiſdem principiis innixas; totius analyſeos ſublimioris fundamentum æquè labefactando.

75. Nec minùs ruit altera iſta objectio *D'Alembert* (*f*), in vocem "*Velocitatem*" animadvertentis, quippè quæ eodem redit in ſenſu *Algebräico*, ac "*ratio:*"

(*f*) "To ſpeak of *D'Alembert* merely as a *mathematician*," THE MONTHLY REVIEW ſeems rather to over-rate his merits:—However profound his reſearches, and extenſive his labours "to clear away the rubbiſh of *jargon* and *myſtery* from ſcience," his calculations and demonſtrations are exceedingly rugged, prolix and intricate; far removed from the elegance, conciſeneſs and arrangement of *Newton*, and even of *Euler* and *Clairaut*.—When *Landen* (himſelf a profound mathematician, and who firſt diſcovered the ſource of *Newton*'s miſtake, in determining the quantity of the *preceſſion of the equinoxes*, the merit of which is attributed to *D'Alembert* by the M. R.) had examined *D'Alembert*'s ſolution of the *Theory of Rotatory Motion*, differing from his own, he was unable to diſcover the ſource of that difference in the *obſcurity* of his argument, until aſſiſted by the perſpicuity of *Euler*'s manner; whoſe ſolution agreed in it's reſult with *D'Alembert*'s.—And ſurely his objection to the Method of Fluxions ſhews that he was not deeply verſed in the *metaphyſique* of the ſcience.

et in ſenſu *Geometrico*, legitimè pro *fluxione* uſurpanda eſt, cum *velocitas* ſit "*exponens rationis*" inter ſpatium et tempus intercedentis, ubi motus ſit uniformis; et proinde *relationem* verè exiſtentem inter veras quantitates commenſurabiles in *Mechanicâ Rationali*, aptè, et "*naturalitèr* quidem," (ipſo fatente M. R. p. 498.) deſignat. Et ſecundùm *D'Alembert—Calculi Differentialis* in Geometricis, munus eſt—"*déterminer algebräiquément* LA LIMITE *d'un rapport de laquelle on à déjà l'expreſſion en lignes; et à égaler ces* deux limites; *ce qui fait trouver* une des lignes *qu'on cherche*."

Atque illuſtrandi gratiâ, ductâ tangente *parabolæ*, cujus æquatio eſt $ax = y^2$, primò exquirit *D'Alembert* valorem unius limitis algebräicè, $(\frac{a}{2y})$; et valorem alterius geometricè $(\frac{y}{s})$; quibus æquatis, deducit valorem *ſubtangentis*, $(s) = \frac{2y^2}{a} = 2x$. Atque agnoſcit paritèr, (ut, ex abundanti diluatur objectio *Torelliana*) quòd in formulâ $\frac{dx}{dz}$, ſecundùm notationem differentialem, ſeu $\frac{\dot{x}}{\dot{z}}$ ſecundùm fluxionalem), *dx* et *dz* (ſeu $\dot{x}$ et $\dot{z}$) non deſignare *quantitates infiniteſimas*, ſeu infinitè parvas, ſed in certo quodam ſtatu *finitæ magnitudinis*; tales enim *ſymbolicè* tantùm uſurpari pro formâ loquendi compendioſâ, calculis concinnandis accommodatâ, verùm nullatenùs *criticè*, ſeu pro *uſu philoſophico* terminorum technicorum.

76. Atque hoc pacto, ipſe quoque *Cenſor*, p. 487, monet, quòd ſi *curvæ cujuſdam axis* denotet *tempus* deſcriptionis; et *ordinatìm applicatæ* quibuſvis curvæ ipſius punctis, *ſpatia* (vel actu deſcripta vel quæ deſcribenda forent velocitate exinde uniformitèr continuatâ); erit *velocitas* ubivis, ut *quotiens ordinatæ per ſubtangentem diviſæ*, ſeu $v = \frac{o}{s}$: et proindè per hanc rationem *verè* exponitur ſeu menſuratur.

77. Hactenùs explicatis elementis primis Methodi Fluxionum, pro ſuâ ſubtilitate fuſiùs; facilitati conceptûs conſulens, et ſpeculationibus *metaphyſicis* facem præbens, quantum res difficillima pateretur; et abſolutis *vindiciis* adverſùs obtrectatores laudum *Newtonianarum*, tàm exteros quàm noſtrates; gratiam ut ſpero, promeriturus *omnium* cordatiorum analyſtarum, qui quantum *Newtono* debent agnoſcere haud gravantur; (*Anglorum* potiſſimùm, ingens illud decus BRITANNIÆ, atque Analyſeos ſublimioris principem et architectum, optimo jure admirantium): Supereſt ut ipſius methodi generalis *rudimenta* proponam; preſſiùs atque concinniùs, et tamen uberiùs ac luculentiùs, quàm

alii ſcriptores qui hactenùs hoc facere aggreſſi ſunt, hâc, nimirùm, lege mihi propoſitâ et ſedulò obſervatâ, ut veſtigiis ipſius inventoris ſagaciſſimi, quàm maximè potuerim, fidelitèr ſemper inſiſterem ;

Te ſequor, O MAGNÆ GENTIS DECUS ! *inque tuis nunc*
Fixa pedum pono preſſis veſtigia ſignis ;
Non ita certandi cupidus, quàm propter amorem,
Quòd te imitari aveo :——
E tenebris tantis tam clarum extollere lumen
Qui primus potuiſti, illuſtrans commoda vitæ.—
Tu PATER ! ET RERUM INVENTOR !——
————*Tuíſque ex*, INCLUTE, *chartis*,
Floriferis ut APES *in ſaltibus omnia libant*,
Omnia NOS *itidem depaſcimur aurea dicta*,
Aurea, perpetuâ ſemper digniſſima vitâ. LUCRETIUS.

Nam æquum certè erit has laudes ad *Newtonum* transferre, VIRTUTIS VERÆ cuſtodem, SAPIENTIÆ rigidum ſatellitem, quas de *Epicuro*, “ *inſanientis ſapientiæ* ” *patre* et “ *patrono* ” falsò prædicat *Lucretius* ; qui ipſe, vitæ pertæſus, non ultrà quadrageſimum ætatis annum provectus, necem ſibi conſcivit.

MISERÆ PHILOSOPHIÆ MISERUM SOLAMEN !

ANALYSIS FLUXIONUM.

PARS SECUNDA.

DEFINITIONES.

§ 78. 1. METHODUS *Fluxionum* eſt quæ determinat quantitatum ejuſdem generis, utcunque variabilium, rationes ad ſe invicèm, quolibet in ſtatu contemporaneo variationis ſeu mutationis, per *methodum primarum et ultimarum rationum* ſeu *limitum*.

2. *Rationes ultimæ* ſunt *limites*, ad quos quantitatum ſine fine decreſcentium *rationes*, 1, *ſemper appropinquant*; et, 2, quas *propiùs aſſequi* poſſunt quàm *pro datâ quâvis differentiâ*; 3, *nunquam verò tranſgredi*; 4, *nec priùs attingere*, quàm quantitates ipſæ diminuuntur in infinitum. Et par eſt natura *rationum primarum*.

3. *Fluens* eſt quantitas ipſa variabilis, ſed tamen finita, quemvis mutationis ſtatum ſubiens, antequam evaneſcat, ſeu in infinitum augeatur.

4. *Genita* eſt ipſius fluentis magnitudo *certo* quodam variationis puncti, ſeu mutationis ſtatu.

5. *Momentum* eſt fluentis augmentum aut decrementum *momentaneum*; id eſt, tempore quam minimo genitum. Eſtque *fluxioni* proportionale.

6. *Fluxio*, ſeu *Functio Analytica*, ſeu *Differentialis*, eſt *menſura quævis rationis ſecundùm quam variatur fluens, quolibet in ſtatu mutationis*.

7. Fluentis ipſius *Fluxio* appellatur *prima*, ſeu *primi ordinis*; hujus iterùm *fluxio* (ſi prima fluxio variabilis ſit) *ſecunda*, ſeu *ſecundi ordinis*; (ſi ſecunda quoque ſit variabilis,) hujus denuò *fluxio*, *tertia*, ſeu *tertii ordinis*; atque ita deinceps, donec ultimò perveniatur ad *fluxionem conſtantem*, ſeu mutatione uniformi procedentem.

8. *Methodus Fluxionum Directa*, quæ *ex fluentibus datis invenit fluxiones*; *Methodus Inverſa*, viciſsìm, *ex fluxionibus datis invenit fluentes*.

9. *Æquatio Fluentialis* ſeu *Integrans*, quæ ex fluentibus ſolis conſtat; *Æquatio Fluxionalis* ſeu *Differentialis*, quæ fluxiones fluentibus immiſtas continet.

AXIOMATA.

79. 1. *Quantitates*, ut et *Quantitatum Rationes*, quæ ſunt *conſtantes* aut *datæ*, nullas ſortiuntur fluxiones.

2. Quantitatum vel Rationum in *maximis* aut *minimis* verſantium, vel in *limitibus* ipſis, evaneſcunt fluxiones.

3. *Fluxio ſummæ* duarum pluriúmve fluentium æquatur *ſummæ fluxionum* ſingularum fluentium cum propriis ſignis; ſcilicet, affirmativis, ſi momenta pro incrementis habeantur; negativis, ſi pro decrementis.

4. Quantitates quæ *conſtantèr*, ſeu *æquabilitèr*, ſeu *certâ* et *determinatâ quâdam ratione* fluunt, fluxiones tantùm *primas* ſortiuntur.

5. Fluxio cujuſvis fluentis ſive puræ, ſive cum cognitis quibuſcunque quantitatibus immiſtæ, eſt eadem. Sic, ſi A ſit quantitas variabilis, et *e* quantitas data, quantitatis A et A ± *e* eadem erit fluxio *a*.

6. Eadem fluens *plures* ſortiatur fluxiones; ſi modo ſint in datâ ad invicem ratione. Sic quantitatis $A^{\frac{m}{n}}$, cujus prima fluxio eſt $\frac{m}{n} a A$, ſecunda erit $\frac{m}{n} \cdot \frac{m}{n} - a^2 A^{\frac{m}{n}-2}$; vel huic proportionalis, $\frac{m}{n} \cdot \frac{\frac{m}{n}-1}{2} a^2 A^{\frac{m}{n}-2}$; nempè, in ratione datâ 1 ad $\frac{1}{2}$.

Pauca inſupèr, *præparationis* loco, ſubjungere libet de *variis formis notationi* quæ in calculo ſive fluxionali ſive differentiali adhiberi ſolent.

De Notatione Fluxionali.

80. *Fluentes* primò deſignabat *Newtonus*, in *Lem.* II. *Lib.* II. *Princip.* per literas majuſculas A, B, C, &c. Et harum *momenta*, (ſeu *fluxiones* momentis proportionales), per minuſculas *a*, *b*, *c*, &c.; quæ relationes quantitatum ipſarum reſpectu magnitudinis benè exponunt. Poſteà verò, alteram illam methodum, jam uſitatiorem, excogitavit, deſignandi fluentes per minuſculas *z*, *y*, *x*, &c. harúmque fluxiones diverſorum ordinum per puncta ſuper-impoſita, hoc modo:

$\dot{z}$, $\dot{y}$, $\dot{x}$ &c - - - *fluxiones primæ*,
$\ddot{z}$, $\ddot{y}$, $\ddot{x}$ &c - - - *fluxiones ſecundæ*,
$\dddot{z}$, $\dddot{y}$, $\dddot{x}$ &c - - - *fluxiones tertiæ*,
$\ddddot{z}$, $\ddddot{y}$, $\ddddot{x}$ &c - - - *fluxiones quartæ*.

81. Secundùm *notationem differentialem*, minùs elegantèr exponuntur *differentiales* uniuſcujuſque gradus,

dx (ſeu *xi*), - *differentialis prima*,
ddx, - - - *differentialis ſecunda*,
dddx, - - - *differentialis tertia*,
ddddx, - - - *differentialis quarta*.

Secundùm notationem quam uſurpat *De La Grange*, ſi curvæ cujuſvis æquatio ſit $y = fx$, erunt valores, *ſubnormalis*, *ſubtangentis* et *radii curvaturæ* yy^{I}, $\frac{y}{y^{\mathrm{I}}}$, et $\frac{(1+y^{\mathrm{I}2})^{\frac{3}{2}}}{y^{\mathrm{II}}}$, reſpectivè, quæ ſecundùm notationem Newtonianam prodeunt $\frac{y\dot{y}}{\dot{x}}$, $\frac{y\dot{x}}{y}$, et $\frac{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}{x\ddot{y}}$.—*Cenſore* referente.

In *tertiâ editione Princip.* p. 486, deſignat quoque *Newtonus differentias primas* per *b*, 2*b*; *ſecundas*, per *c*, 2*c*; *tertias*, per *d*, 2*d*, &c.

82. Ingenti autèm Matheſeos commodo, introduxit Newtonus in *Algebram Specioſam* notationem *indicum fractorum* et *negativorum*; quarum rationem brevitèr exponere in animo eſt, ne confundantur tyrones notatione inuſitatâ;

præſertìm, cùm non deſint auctores, qui uſum indicum *fractorum* et *negativorum* pravâ confuſione miſcent.

Indices *integri* quantitatum A^2, A^3, A^4, A^m, indicant varias *poteſtates* ſeu dignitates, per *involutionem* reſultantes. Indices *fracti* $A^{\frac{1}{2}}$, $A^{\frac{1}{3}}$, $A^{\frac{1}{4}}$, $A^{\frac{1}{n}}$, varias itidem *radices*, per evolutionem; quales nempè referunt *ſurdæ* quantitates, ſecundùm notationem vulgarem, $\sqrt{A}$, $\sqrt[3]{A}$, $\sqrt[4]{A}$, $\sqrt[n]{A}$. Ex utriúſque autem combinatione, emergent $A^{\frac{2}{3}}$, $A^{\frac{3}{4}}$, $A^{\frac{5}{2}}$, $A^{\frac{m}{n}}$, deſignantes nimirùm radices per denominatores expoſitas, poteſtatum per numeratores, $\sqrt[3]{A^2}$, $\sqrt[4]{A^3}$, $\sqrt{A^5}$, $\sqrt[n]{A^m}$. Indices verò *negativi*, A^{-2}, A^{-3}, A^{-m}, $A^{-\frac{2}{3}}$, $A^{-\frac{3}{4}}$, $A^{-\frac{m}{n}}$, indicant *reciprocos* affirmativorum, $\frac{1}{A^2}$, $\frac{1}{A^3}$, $\frac{1}{A^m}$, $\frac{1}{A^{\frac{2}{3}}}$, $\frac{1}{A^{\frac{3}{4}}}$, $\frac{1}{A^{\frac{m}{n}}}$.

Atque hoc pacto, *fractiones tolli*, et *æquationes concinnari* poſſunt, tranſponendo quantitates indicibus negativis inſignitas, à numeratore in denominatorem; aut vice verſâ.

Sic $\frac{A^1}{A^2} = \frac{1}{A^2} = A^{-2}$; et $\frac{A^m}{A^n} = \frac{1}{A^{n-m}} = A^{m-n}$; et, $A^{\frac{1}{4}-3} = \frac{1}{A^{3-\frac{1}{4}}}$ $= \frac{1}{A^{2+\frac{3}{4}}}$; et, $A^{\frac{m^2}{n^2}-3} = \frac{1}{A^{3-\frac{m^2}{n^2}}}$, et, ſic, $\frac{naA^{n-1}}{A^n} \times \frac{1}{A^n} = \frac{na}{A} \times \frac{1}{A^n} =$ $\frac{na}{A^{n+1}} = naA^{-n-1}$.

De Methodo Fluxionum Directâ.

83. Continentur canones, ſeu methodi hujúſce regulæ, in theoremate quodam generaliſſimo, omnes caſus comprehendente:

LEMMA II.

" *Momentum genitæ* ($A^m B^n$) *æquatur momentis* (i. e. *ſummæ momentorum*, ſcilicet ($a + b$) *laterum ſingulorum generantium* (A, B) *in eorundem laterum indices dignitatum* (m, n) *et co-efficientia* ($A^{m-1}B^n$, $B^{n-1}A^m$), *continuè ductis*." Seu fluxio $A^m B^n = maA^{m-1}B^n + nbB^{n-1}A^m$.

N. B. Phraſi

N. B. Phraſi Newtonianâ, "*Lateris cujuſque generantis co-efficiens, eſt quantitas quæ oritur, applicando* [*totam quantitatem*] *genitam ad hoc latus.*" Sic lateris A *co-efficiens* eſt $\frac{A^m B^n}{A} = A^{m-1} B^n$; et lateris B, eſt $\frac{A^m B^n}{B} = B^{n-1} A^m$.

84. Auſpicatur *Newtonus*, à caſu ſimpliciſſimo.

Caſ. 1. *Fluxio rectanguli geniti* (AB) *æquatur fluxionibus laterum generantium* (*a* et *b*) *in ipſa latera* (B et A) *continuè ductis*; ſeu fluxio AB $= aB + bA$.

Rectanguli cujuſvis perpetuò fluentis, ſint latera, vel æquabilitèr vel pari inæquabilitate creſcentia, per datam quandam temporis particulam, in *medio ſtatu* mutationis, A et B. Sumantur horum ſemi-momenta antecedentia, pro decrementis, $-\frac{1}{2}a$ et $-\frac{1}{2}b$; conſequentia autèm, pro incrementis, $+\frac{1}{2}a$ et $+\frac{1}{2}b$. Laterum igitùr variabilium prodibunt valores ſucceſſivi, eorúmque differentiæ correſpondentes à mediis:

$$\begin{array}{llll}
\mathit{Val.}\ \mathrm{A.} & \mathit{Diff.} & \mathit{Val.}\ \mathrm{B.} & \mathit{Diff.} \\
\left.\begin{array}{l} A - \tfrac{1}{2}a \\ \\ A \quad - \end{array}\right\} & \begin{array}{l} - \;-\; - \\ + \tfrac{1}{2}a, \\ - \;- \;\text{et} \end{array} & \left.\begin{array}{l} B - \tfrac{1}{2}b \\ \\ B \quad - \end{array}\right\} & + \tfrac{1}{2}b. \\
A + \tfrac{1}{2}a & \left\{\dfrac{+\tfrac{1}{2}a}{+a}\right. \quad - & B + \tfrac{1}{2}b & \left\{\dfrac{+\tfrac{1}{2}b}{+b}\right..
\end{array}$$

Horum valorum primi erunt minimi, et poſtremi, maximi; ſed fluxiones laterum ſunt eorum incrementis *totis* ſimultaneis, *a* et *b*, proportionales reſpectivè; et proinde jure exponentur per ſummas differentiarum, $a + b$.

Rectanguli quoque variabilis, prodibunt valores ſucceſſivi, eorúmque differentiæ, à medio valore, AB:

$$\begin{array}{ll}
\qquad\qquad\qquad\qquad \mathit{Val.} & \mathit{Diff.} \\
\left.\begin{array}{l} (A - \tfrac{1}{2}a) \times (B - \tfrac{1}{2}b) = AB - \tfrac{1}{2}aB - \tfrac{1}{2}bA + \tfrac{1}{4}ab \\ A \times B \;-\;-\;-\;- = AB \;-\;-\;-\;- \end{array}\right\} & + \tfrac{1}{2}aB + \tfrac{1}{2}bA - \tfrac{1}{4}ab \\
(A + \tfrac{1}{2}a) \times (B + \tfrac{1}{2}b) = AB + \tfrac{1}{2}aB + \tfrac{1}{2}bA + \tfrac{1}{4}ab & \left\{\dfrac{+ \tfrac{1}{2}aB + \tfrac{1}{2}bA + \tfrac{1}{4}ab}{+ aB + bA\ *}\right..
\end{array}$$

(*g*) "*Quantitates differentiis ſuis proportionales ſunt continuè proportionales.*" Et converſìm *quantitatum continuè proportionalium erunt differentiæ ipſis quantitatibus proportionales.*

1. Sit A ad A — B ut B ad B — C, et C ad C — D, &c. Et *convertendo* fiet A ad B ut B ad C, et C ad D.

2. Sit A ad B ut B ad C, &c. Et iterùm fiet A ad A — B ut B ad B — C.

Vide *Maclaurin's Fluxions*, § 159—160, ubi prolixè demonſtratur et Theoriæ Fluxionum Logarithmorum applicatur.

Et horum valorum quoque, erit primus minimus, et poſtremus, maximus. Sed fluxio rectanguli incrementis laterum *totis* a et b geniti, erit incremento toti proportionalis; et proindè jure exponetur per ſummam differentiarum, $aB + bA$.

Si negas, fingatur fluxionem veram eſſe vel majorem vel minorem, quantitate quâvis aſſignabili D, ſeu, $aB + bA \pm D$. Quod ſi major foret, emergere *poſſit* dimidium novæ fluxionis, $\frac{aB + bA + D}{2}$ majus maximâ, (per conſtructionem) differentiarum, $\frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab$; et proindè non ritè exprimeret augmentum rectanguli poſt medium ſtatum AB: ſin minor, emergere *poſſit* dimidium novæ fluxionis $\frac{aB + bA - D}{2}$, minus minimâ differentiarum, $+ \frac{1}{2}aB + \frac{1}{2}bA - \frac{1}{4}ab$; et proindè non ritè exprimeret augmentum rectanguli ante medium ſtatum AB. Q. E. D.

In numeris res patebit:

Sit $A = 20$, et $a = 2$; $B = 30$, et $b = 4$; erunt

$$\begin{array}{lr}
 & \textit{Val.}\quad \textit{Diff.}\\
\left.\begin{array}{l}(A - \frac{1}{2}a) \times (B - \frac{1}{2}b) = 19 \times 28 = 532\\ A \times B \quad - \; - \; - \quad = 20 \times 30 = 600\end{array}\right\} & 68\\
(A + \frac{1}{2}a) \times (B + \frac{1}{2}b) = 21 \times 32 = 672\,\} & \underline{72}\\
 & 140.
\end{array}$$

Sed $aB + bA = 60 + 80 = (140 =)\ 68 + 72$.

85. Demonſtrationem totam breviſſimè expedit *Newtonus:*

"Rectangulum quodvis *motu perpetuo* auctum AB, *ubi* de lateribus A et B deerant momentorum dimidia $\frac{1}{2}a$ et $\frac{1}{2}b$, fuit $A - \frac{1}{2}a$ in $B - \frac{1}{2}b$, ſeu $AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab$; et *quàm primùm* latera A et B alteris momentorum dimidiis aucta ſunt, evadit $A + \frac{1}{2}a$ in $B + \frac{1}{2}b$, ſeu $AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab$. De hoc rectangulo ſubducatur rectangulum prius, et manebit *exceſſus* $aB + bA$. —Igitùr laterum incrementis *totis* a et b, generatur rectanguli incrementum $aB + bA$." Q. E. D.

In hâc demonſtratione, *Newtonus*, neglecto *medio* valore rectanguli fluentis, nempè AB, *ſimùl ac ſemèl*, ſubducit minimum valorem de maximo; et inde, *exceſſus* deſignabit totum rectanguli augmentum *totis* momentis a et b generatum, (ſive *ſummam* differentiarum minimi valoris à medio, et medii à maximo); ſed *argumentum ad abſurdum* omittit, quaſi ex ſuis principiis in *Lemmate primo* ſatìs manifeſtum. Vid. § 16. Not. (*c*).

86. Demonſtrationis *Newtonianæ* vim atque ſubtilitatem minimè capiens "*Analyſta*" (*Berkeley,*) validitatem ejus imperitè improbavit. Nec ſatis perſpexerunt vindices, *Colſon* et *Maclaurin:* *Colſon* enim *argumentum ad abſurdum*, propoſitioni plenè, perfectè et rigidè demonſtrandæ neceſſarium, omiſit; *Maclaurin* verò viciſsìm, priorem demonſtrationis partem, à *Colſon* benè explicatam,

paritèr omiſit; huic parti poſteriori tantùm immorans, prolixiùs et obſcuriùs, in demonſtratione parallelâ fluxionis *quadrati*; unde, ordine inventoris lucido in pejus mutato, fluxionem *rectanguli* deducit; *non benè relictâ* interpretis *parmulâ:* ſicùt in *Parte Primâ* monuimus, § 16.

87. Analyſtæ recentiores quoque, *Euler*, (§ 72.) &c. ſolidam et rigidam demonſtrandi viam *Newtonianam* deſerentes, rem paulò alitèr inſtituunt, ex *methodo infiniteſimalium*.

Laterum generantium ſint poſtremi valores, $x + \acute{x}$ et $y + \acute{y}$; emerget poſtremus quoque valor rectanguli ſimultanei, his in ſe ductis, $xy + \acute{x}y + x\acute{y} + \acute{x}\acute{y}$. De hoc valore ſubducatur primus, xy, et manebit *exceſſus* $\acute{x}y + x\acute{y} + \acute{x}\acute{y}$; nempè incrementis $\acute{x}$ et $\acute{y}$ generatus; cæterùm terminus ultimus $\acute{x}\acute{y}$, utpotè *infinitè parvus*, tutò *negligi* poteſt: et proindè incrementum ipſius rectanguli quàm proximè exponet $\acute{x}y + x\acute{y}$; ſive ſubſtitutis, pro incrementis $\acute{x}$ et $\acute{y}$, fluxionibus $\dot{x}$ et $\dot{y}$, fluxionem rectanguli exponet $\dot{x}y + x\dot{y}$. Q. E. D.

Vide *Hutton's Mathematical Dictionary*, voce *Fluxion*, p. 487; undè hanc demonſtrationem *Newtonianæ* (ibidem malè omiſſæ) evidentiâ et elegantiâ longè cedentem, deprompſimus: Nam fluxionem rectanguli variabilis, quam in ſtatu mutationis *primo*, xy, non niſi *quam proximè* exponit $\dot{x}y + x\dot{y}$; in ſtatu *medio*, ſeu *limite* in ipſo, AB, longè accuratiùs exponit $a\mathrm{B} + b\mathrm{A}$, ſecundùm conſtructionem *Newtonianam* artificioſiorem.

88. Atque ex hujus demonſtrationis ſubtiliſſimæ genio ſeu indole, *methodum limitum* indubitatò reſpiciente, optimè forsàn explicare licet principium illud ſpinoſiſſimum et vexatiſſimum, ſed nobiliſſimum et generaliſſimum ſanè, unde pendet omnis theoria *Newtoniana* virium centripetarum, *Princip. Lib.* I. *Prop.* 1. *Cor.* 3. et 4.

Cor. 3. "*Vires centripetæ* in [*mediis* arcuum momentaneorum ABC et DEF] B et E, ſunt ad invicèm in *ultimâ ratione diagonalium* BV, EZ, [*quæ convergunt ad centrum virium* S] *ubi arcus iſti in infinitum diminuuntur.*"

Nam vires centripetæ, in his mediis locis B et E, utpote *acceleratrices*, propriè menſurantur per *velocitates* versùs commune centrum motûs cadendo acquiſitas, æqualibus his temporibus quàm minimis; atque hæ iterùm *velocitates genitæ*, utpote temporum quam minimorum decurſu, ferè æquabiles, ob hæc tempora, per hypotheſin, æqualia, erunt ut *ſpatia*, impulſibus vis centripetæ in locis B et E interim generata versùs commune centrum motûs S; ſive *deflexiones* de extremis *c* et *f* tangentium per B et E ductarum; cæterùm hæ deflexiones *ferè* (*b*) coincident cum *ſubtenſis* angulorum contactûs *c*C, *f*F, *ubi arcus iſti*

(*b*) Vide *La Caille, Elémens d'Aſtronomie*, ſeu Verſionem Anglicanam à *Robertſon*, § 157, et fig. 33. ubi gradus hic demonſtrationis præcipuus, à *Newtono* omiſſus, et à commentatoribus malè neglectus, optimè ſuppletur: ibi enim notantur *hæ deflexiones veræ* verſùs centrum per QR, *p*F, quæ cum *ſubtenſis* angulorum contactûs, radio vectori parallelis reſpectivè, *ferè* coincidunt.

ABC et DEF *in infinitum diminuuntur* ; (nimirùm recedendo utrinque ab extremis A et C, atque D et F, versùs media B et E, reſpectivè) ; itémque hæ ſubtenſæ cC, fF ;—ſi reſolvatur totus revolutionis motus per *ſemiarcus* BC et EF, (qui utique cum *chordis* ſuis et cum *tangentibus* ſuis Bc et Ef ultimò coincident, per *Lem.* 7.) binos in motus, unum, ſecundùm directiones tangentium, et alterum, ſecundùm directiones radiorum vectorum BS, et ES,—erunt parallelæ et æquales *diagonalibus* BV et EZ, quæ convergunt ad centrum virium et chordas AC, DF biſecant : ergò *vires*, &c. erunt *ipſis diagonalibus* proportionales. Q. E. D.

In hâc demonſtratione, (quam, breviſſimè à *Newtono* abſolutam, fuſiùs explicuimus,) quis non videt quod " *ubi arcus iſti in infinitum diminuuntur*," nimirùm recedendo utrinque, &c. ſemiarcus priores AB, DE antè media puncta B et E, *decrementis* $-\frac{1}{2}a$, $-\frac{1}{2}b$; et ſemiarcus poſteriores BC, EF, *incrementis* $+\frac{1}{2}a$, $+\frac{1}{2}b$, reſpondent reſpectivè ?

89. *Cor.* 4. " *Vires illæ centripetæ*, &c. ſunt quoque proportionales *ſagittis illis* quæ chordas biſecant arcuum quorumvis, *æqualibus* temporibus quam minimis deſcriptorum, et convergunt ad centrum virium."

Nam hæ ſagittæ ſunt *ſemiſſes diagonalium* prædictarum, ſeu $\frac{1}{2}$BV, et $\frac{1}{2}$EZ ; et proinde pro ipſis diagonalibus, (*veris* utique virium exponentibus,) compendii gratiâ, calculo cautè inſtituto, et ſervatâ ubique *ſemiſſium proportione*, per totum computum, tutò ſubſtitui poſſunt. Alitèr enim ſuboriri poſſunt errores in calculo minimè contemnendi. Vide *Maclaurin's Fluxions*, § 440 ; *Simſon's Fluxions*, Vol. I. p. 237 ; *La Lande*, *Aſtronomie*, § 3392 ; et *Milner*, *Philoſoph. Tranſ.* anno 1779, *Part* II. p. 521.—Vide plura, § 103, et anteà § 62 et 63.

90. Quò magìs eluceant evidentia, certitudo et ubertas *primi hujus caſûs*, quem totius methodi generalis fundamentalem conſtituit *Newtonus*, placet inſupèr ex conſiderationibus *Geometricis*, abſtractiùs et ſubtiliùs hactenùs tradita, illuſtrare.

1. Lateribus generantibus, A et B, *uniformitèr* creſcentibus, ſive in eâdem, ſive in datâ quâdam ratione ad ſe invicèm ; erit *diagonalis* rectanguli duplici motu geniti *recta linea.* Et *complementa* parallelogrammorum circà diagonalem ritè exponent fluxiones laterum reſpectivè ; adeóque horum aggregatum fluxionem ipſius rectanguli geniti. Sed in hoc caſu ſimpliciſſimo, erunt ipſa complementa æqualia, per *Prop.* 43. *Lib.* I. *Elem.* ; ſive aB $=$ bA, adeóque areæ rectanguli fluxio $=$ 2aB vel 2aA. Et ſi latera ſint æqualia, erit fluxio *quadrati* $=$ 2aA.

2. Lateribus *difformitèr* creſcentibus, ſeu in diverſâ ab invicèm ratione ; ex. gr. ſi momentum a ſit conſtans, at momentum b variabile, erit *diagonalis* rectanguli geniti *curva linea :* quæ erit *convexa* à parte baſeos ſeu abſciſſæ motu conſtanti deſcriptæ, ſi b creſcat ; *concava* autèm, ſi decreſcat. In utróque caſu, complementa illa quæ fluxiones laterum exponunt, erunt inæqualia.

Hinc forſan *Sectiones Conicæ*, nomina ſua propria acceperunt: *Hyperbola*, quaſi motu ordinatæ ὑπερβαλλοντι, "*redundante*," genita; *Ellipſis*, quaſi motu ελλειποντι, "*deficiente*;" et *Parabola*, παραβαλλοντι, ſeu motum ad *conſtantem* ſeu æquabilem *accedente*, genita. Atque his definitionibus nominum affectiones ſingularum ſectionum probè reſpondent.

3. Creſcente uno latere et decreſcente altero, ſive *conſtantèr*, ſive *diverſimodè*, ob momentum poſterioris jam negativum, $-b$, rectanguli fluxio fiet reſidua, $a\text{B} - b\text{A}$.

Caſu hoc primo, omnium fundamentali, jam rigidè demonſtrato ex principiis *purè Algebräicis*, omni *motûs formalis* ſeclusâ conſideratione; et fuſiùs illuſtrato ex *Mechanicâ Rationali* atque *Geometriâ*, ſecundùm mentem ipſius inventoris; reliquos caſus breviùs expedire fas erit; quibuſdam aliundè intertextis.

91. *Caſ.* 1. *Cor.—Hinc fluxio fractionis* $\frac{\text{A}}{\text{B}}$ *fiet* $\frac{a\text{B} - b\text{A}}{\text{B}^2}$. Ponatur $\frac{\text{A}}{\text{B}} =$ Q: erîtque $\text{A} = \text{BQ}$. Adeóque per *Caſ.* 1. $a = q\text{B} + b\text{Q}$; ſive $q = \frac{a}{\text{B}} - \frac{b\text{Q}}{\text{B}} = \frac{a}{\text{B}} - \frac{b}{\text{B}} \times \text{Q} = \frac{a}{\text{B}} - \frac{b}{\text{B}} \times \frac{\text{A}}{\text{B}} = \frac{a}{\text{B}} - \frac{b\text{A}}{\text{B}^2} = \frac{a\text{B}}{\text{B}^2} - \frac{b\text{A}}{\text{B}^2} = \frac{a\text{B} - b\text{A}}{\text{B}^2}$. Q. E. D.

92. *Fluxio contenti ſub lateribus quotcunque*, A, B, C, &c. *æquatur ſummæ ex fluxionibus ſingulorum in reliqua latera ſigillatim continuè ductis.* Sic fluxio ABC, &c. $= a\text{BC}$, &c. $+ b\text{AC}$, &c. $+ c\text{AB}$, &c.

Ponatur $\text{AB} = \text{G}$. Et per *Caſ.* 1. erit fluxio ABC, ſeu $\text{GC} = g\text{C} + c\text{G} = (a\text{B} + b\text{A}) \times \text{C} + c \times \text{AB} = a\text{BC} + b\text{AC} + c\text{AB}$. Et par eſt ratio contenti ſub lateribus quotcunque.

93. *Caſ.* 3. *Fluxio dignitatis cujuſvis affirmativæ* (A^m) *æquatur indici illius dignitatis* (m), *fluxioni lateris* (a), *ejúſque co-efficienti* (A^{m-1}), *in ſe continuè ductis.* Seu, fluxio $\text{A}^m = ma\text{A}^{m-1}$.

Fluxio quadrati A^2, ſeu $\text{A} \times \text{A}$, erit $a\text{A} + a\text{A} = 2a\text{A}$, per *Caſ.* 1. Fluxio itèm cubi A^3, ſeu $\text{A} \times \text{A} \times \text{A}$, erit quoque $a\text{A}^2 + a\text{A}^2 + a\text{A}^2 = 3a\text{A}^2$, per *Caſ.* 2. Et, eodem argumento, fluxio $\text{A}^m = ma\text{A}^{m-1}$. Q. E. D.

Cor. Hinc fluxio $\frac{\text{A}^m}{m}$ erit $a\text{A}^{m-1}$.

Scholium. Proprietate curvarum elegantiſſimâ, ſi area quævis curvilinea ſuper *baſi* (AP) deſignetur per A^m, quolibet in ſtatu mutationis; ſemper exponetur *ordinata* (PM) per maA^{m-1}. Vide *Maclaurin's Fluxions*, § 936, fig. 352.

94. *Caſ.* 4. *Fluxio dignitatis cujuſvis negativæ* (A^{-m}) *æquatur indici, in lateris fluxionem ductæ* ($-ma$), *et per indicem genitæ unitate auctum diviſæ* ($m+1$). Seu fluxio $A^{-m} = \frac{-ma}{A^{m+1}}$.

Quantitatis conſtantis $A \times \frac{1}{A} = 1$, erit fluxio nihil, per *Ax.* 1. Fiat $\frac{1}{A} = B$; erítque fluxio $AB = aB + bA = 0$. Adeóque b, ſeu fluxio $\frac{1}{A}$ vel $A^{-1}) = \frac{-aB}{A} = \frac{-a}{A} \times \frac{1}{A} = \frac{-a}{A^2}$. Et generalitèr, quantitatis conſtantis $A^m \times \frac{1}{A^m} = 1$, fluxio erit nihil. Fiat $\frac{1}{A^m} = B$, erítque fluxio $A^m \times B = bA^m + maBA^{m-1} = 0$. Adeóque b, (ſeu fluxio $\frac{1}{A^m}$ vel A^{-m}) $= \frac{-maBA^{m-1}}{A^m} = \frac{-maB}{A^1} = \frac{-ma}{A^1} \times \frac{1}{A^m} = \frac{-ma}{A^{m+1}}$. Q. E. D.

95. *Caſ.* 5. *Fluxio cujuſcunque dimenſionis fractæ affirmativæ* ($A^{\frac{m}{n}}$ *æquatur indici illius dimenſionis* ($\frac{m}{n}$), *lateris fluxioni* (a), *ejúſque co-efficienti* ($A^{\frac{m}{n}-1}$), *in ſe continuè ductis.* Seu fluxio $A^{\frac{m}{n}} = \frac{m}{n} aA^{\frac{m}{n}-1}$.

Quantitatis $A^{\frac{1}{2}} \times A^{\frac{1}{2}} = A$ fluxio erit a. Fiat $A^{\frac{1}{2}} = B$; adeóque fluxio B^2 (ſeu $A^{\frac{1}{2}} \times A^{\frac{1}{2}}$) erit $2bB = a$; et proinde b (ſeu fluxio $A^{\frac{1}{2}}$) $= \frac{a}{2B} = \frac{a}{2A^{\frac{1}{2}}}$. Et generalitèr: fiat $A^{\frac{m}{n}} = B$; erítque $A^m = B^n$; et proinde harum fluxiones erunt inter ſe æquales, nempè $maA^{m-1} = nbB^{n-1}$; et dividendo utrinque per $A^m = B^n$, erit $maA^{-1} = nbB^{-1}$; ideóque erit b (ſeu fluxio B, ſive $A^{\frac{m}{n}}$) $= \frac{maA^{-1}}{nB^{-1}} = \frac{maA^{-1}}{n \times A^{-\frac{m}{n}}} = \frac{maA^{\frac{m}{n}-1}}{n}$. Q. E. D.

96. N. B. Pro co-efficientis indice, $\frac{m}{n} - 1$, uſurpat *Newtonus* indicem æquipollentem $\frac{m-n}{n}$ $(= \frac{m}{n} - \frac{n}{n}) = \frac{m}{n} - 1$, in demonſtratione hujus caſûs; unde prodit fluxio $A^{\frac{m}{n}} = \frac{m}{n} aA^{\frac{m-n}{n}}$. Eâdem nimirùm notatione indicum hâc occaſione uſus, ac compluribus ante annis, nempè anno 1676, ubi primò præclarum ſuum *Theorema Binomium* in lucem protulit. Vid. ANALYS. ÆQUAT. § 57. Cæterùm hoc non levem ſuppeditat conjecturam, Methodum Fluxionum, vel illo tempore, Newtono perſpicuè innotuiſſe; ut poſthàc pleniùs et irrefragabiliùs evincetur ex inſigni analogiâ inter terminos ſucceſſivos ſeriei binomiæ, atque fluxiones ſuperiorum ordinum, intercedente; talem enim analogiam inter utramque methodum, utriuſque inventorem effugiſſe, ſuſpicari non fas eſt.

97. *Caſ.* 6. *Fluxio genitæ cujuſcunque ex duobus pluribúſve factoribus, æquatur ſummæ ex fluxionibus uniuſcujuſque factoris in reliquam factorem vel reliquos factores, continuè inter ſe ductis.* Sic fluxio $A^m B^n = maB^n A^{m-1} + nbA^m B^{n-1}$.

Ex præcedentibus conſtat; nempe, *Caſ.* 1 et 3. Et par eſt ratio contenti ſub pluribus factoribus; nam, per *Caſ.* 2, erit fluxio $A^m B^n C^r = maB^n C^r A^{m-1} + nbA^m C^r B^{n-1} + rcA^m B^n C^{r-1}$.

Idem quoque obtinebit per *Caſ.* 5, ubi indices factorum ſunt *fracti*, $\frac{m}{n}$, $\frac{r}{s}$, &c. $A^{\frac{m}{n}} B^{\frac{r}{s}}$; cujus fluxio evadet $\frac{m}{n} aB^{\frac{r}{s}} A^{\frac{m}{n}-1} +$ &c.

Et hoc eſt *Theorema Fluxionale* nobiliſſimum et generaliſſimum, uno intuitu omnes caſus comprehendens, ipſi *Theoremati Binomio*, elegantiâ, concinnitate et dignitatis præſtantiâ vix, et ne vix quidem, cedens.

98. Concluſiones præcedentes nullo negotio ad notationem poſteriorem accommodare licet.

Fluxio *rectanguli* $xy = \dot{x}y + x\dot{y}$.

Fluxio *contenti* - $xyz = \dot{x}yz + x\dot{y}z + xy\dot{z}$.

Fluxio *quoti* - $\frac{x}{y} = \frac{\dot{x}y - x\dot{y}}{y^2}$

Fluxio *quadrati* - $x^2 = 2\dot{x}x$.

Fluxio *dignitatis* $x^m = m\dot{x}x^{m-1}$.

Fluxio *dimenſionis* $x^{\frac{m}{n}} = \frac{m}{n}\dot{x}x^{\frac{m}{n}-1}$

Fluxio *genitæ* - $x^m y^n = m\dot{x}x^{m-1}y^n + n\dot{y}y^{n-1}x^m$, vel,

$= (m\dot{x}y + nx\dot{y})\,x^{m-1}y^{n-1}$, vel,

$= (\frac{m\dot{x}}{x} + \frac{n\dot{y}}{y})\,x^m y^n$.

—Itémque, $x^m y^n z^r = m\dot{x}x^{m-1}y^n z^r + n\dot{y}y^{n-1}x^m z^r + r\dot{z}z^{r-1}x^m y^n$,

vel, $= (m\dot{x}yz + nx\dot{y}z + rxy\dot{z})\,x^{m-1}y^{n-1}z^{r-1}$,

vel, $= (\frac{m\dot{x}}{x} + \frac{n\dot{y}}{y} + \frac{r\dot{z}}{z})\,x^m y^n z^r$.

99. Corollaria quædam *Newtoniana*, à *Maclaurin* omiſſa, hìc adjungere libet, ex præcedente theoriâ reſultantia, eandémque optimè illuſtrantia, et in *Logarithmotechniâ* valdè utilia.

Corol. 1. *Hinc, in continuè proportionalibus, ſi terminus unus datur, fluxiones terminorum reliquorum, erunt ut iidem termini multiplicati per numerum intervallorum inter ipſos et terminum datum.*

Sunto A, B, C, D, E, F, continuè proportionales, et ſi *detur* terminus intermedius C, (cujus idcircò fluxio erit *nihil,*) exquirantur tùm valores reliquorum, ex notis proportionalium proprietatibus, tùm horum fluxiones, ex præcedenti theoriâ, itémque fluxionum rationes:

Proport.	*Valor.*	*Flux.*	1 *Rat.*	2 *Rat.*
A ſive	$\frac{C^3}{D^2}$	$C^3 \times \frac{-2d}{D^3}$	$-\frac{2C^3}{D^2}$	$-2A$
B	$\frac{C^2}{D}$	$C^2 \times \frac{-d}{D^2}$	$-\frac{C^2}{D}$	$-1B$
C	1	0	0	0
D	D	d	$+D$	$+1D$
E	$\frac{D^2}{C}$	$\frac{2dD}{C}$	$+\frac{2D^2}{C}$	$+2E$
F	$\frac{D^3}{C^2}$	$\frac{3dD^2}{C^2}$	$+\frac{3D^3}{C^2}$	$+3F$.

In hâc tabulâ, ex ſerie fluxionum elicitur prima ſeries rationum $-\frac{2C^3}{D^2}$, $-\frac{C^2}{D}$, &c.; multiplicando nimirùm ſingulas fluxiones correſpondentes, per communem quantitatem $\frac{D}{d}$; ſecunda ſeries autèm, ex primâ, reſtituendo pro valoribus ſuis, proportionales ipſos, $-\frac{2C^3}{D^2} = -2A$, &c. Q. E. D.

100. *Corol.* 2. *Et ſi in quatuor proportionalibus duæ mediæ dentur, fluxiones extremarum erunt ut eædem extremæ.*

Nam, ob datum idcircò rectangulum ſub extremis AD (utpote æquale BC), erit fluxio ejus $aD + dA = 0$; ideóque, $aD = -dA$; et proinde, $a : -d :: A : D$. Q. E. D.

Idem intelligendum eſt de lateribus rectanguli cujuſcunque dati. Pari enim ratione, erit $b : -C :: B : C$. Cæterùm in hujuſmodi caſibus, ob fluxiones negativas $-c$, $-d$, diverſimodè variantur latera generantia, uno creſcente, altero decreſcente.

101. *Corol.* 3. *Et ſi ſumma vel differentia duorum quadratorum detur, fluxiones laterum erunt reciprocè ut latera.*

Sint enim quadrata $A^2 \pm B^2 = 1$, ſeu cuivis quantitati datæ; erit fluxio $2aA \pm 2bB = 0$. Adeóque, $2aA = \mp 2bB$; et proinde, $a : \mp b :: B : A$. Q. E. D.

De Fluxionibus Superiorum Ordinum.

102. Si quantitates fluentes ratione conſtanti, æquabili ſeu datâ, ſimùl augentur vel minuuntur, *fluxiones primas* tantùm, ſeu *primi ordinis*, ſortientur. Quòd ſi harum fluxiones ſint quoque variabiles, ſuas iterùm fluxiones ſortientur, *ſecundas* nempè, ſeu *ſecundi ordinis*; atque hæ iterùm ſuas ſortientur fluxiones ſeu *tertias*; atque ità deinceps, donec ultimò perveniatur ad quantitates conſtantèr ſeu æquabilitèr fluentes.

103. Sic in theoriâ *virium centripetarum*, binis viribus conjunctis urgentibus, revolvuntur corpora in orbibus, ſeu trajectoriis quibuſcunque, circà commune centrum virium: nempè, *vi projectili*, quæ conſtantèr et æquabilitèr agit, ſecundùm directiones tangentium ſingulis in orbitæ locis; et *vi centripetâ*, perpetuo motu *variabili*, corpora de tangentibus versùs centrum deflectente; cùm actio

vis projectilis igitùr eſt *conſtans per ſe* (niſi quatenùs per alteram vim *obliquè* agentem, augetur vel minuitur), *primas* tantùm pariet fluxiones; quarum exponentes erunt *ſpatia* perbrevia æqualibus temporibus quàm minimis percurſa, ſeu *arcus* momentanei cum *tangentibus* ſuis fere coincidentes. At *vis centripetæ*, cum accedendo versùs centrum, fortiùs, recedendo à centro, languidiùs operatur, actio erit variabilis, ejúſque exponentes in locis correſpondentibus, non erunt *ipſa ſpatia*, *momentanea* cadendo versùs centrum percurſa, ſi ſiſteretur vis projectilis (cum ipſæ deflexiones erunt variabiles, pro diverſis à centro diſtantiis), ſed *horum dupla*, ſeu ſpatia quæ velocitatibus *ultimis* cadendo acquiſitis deſcriberentur (ſi exindè conſtantèr velocitates forent uniformes), per eadem tempora quam minima. *Vis centripeta*, igitùr, *fluxiones* ſuas *ſecundas* ſortietur; quarum exponentes ſunt *diagonales illæ*, ſuprà memoratæ, quæ ad centrum commune virium convergunt: hæ enim ſunt *duplæ ſagittarum* illarum, quas corpora his momentis liberè cadendo, *actu* deſcriberent. In *circulis* autèm hæ ſagittæ ſunt ipſi *ſinus verſi*.—Atque hæ obſervationes, theoriæ illæ difficillimæ ac ſubtiliſſimæ virium *projectilium* et *centripetarum*, cui innititur omnis ASTRONOMIA NEWTONIANA, explicandæ et illuſtrandæ, haud leve adjumentum forsàn tyronibus ſuppeditabunt.

104. *Fluxiones ſuperiorum ordinum* iiſdem legibus inveniendæ ſunt ac ipſæ *fluxiones primæ*. Sic generalitèr: genitæ cujuſcunque $A^{\frac{m}{n}}$ erit

Fluxio prima, per *Caſ.* 5, - - - - $\frac{m}{n} a A^{\frac{m}{n}-1}$;

Secunda, $-\left(\frac{m}{n}-1\right)\times(a)\times\left(\frac{m}{n} a A^{\frac{m}{n}-2} = \frac{m}{n}\cdot\frac{m}{n}-1\, a^2 A^{\frac{m}{n}-2}\right.$;

Tertia, $-\left(\frac{m}{n}-2\right)\times(a)\times\left(\frac{m}{n}\cdot\frac{m}{n}-1\, a^2 A^{\frac{m}{n}-3}\right)=\frac{m}{n}\cdot\frac{m}{n}-1\,.\,\frac{m}{n}-2a^3 A^{\frac{m}{n}-3}$.

Et ſic deinceps, eâdem ſervatâ lege derivationis. Ut, ſi cubi A^3 quærantur fluxiones: deſignante $\frac{m}{n}$, 3; erit *fluxio prima*, - - $3aA^2$;

ſecunda, - $6a^2A$;

tertia, - - $6a^3$.

Fluxio autèm *quarta* erit *nihil*; jam evaneſcente factore A.

Hinc, ſi genitæ index ſit numerus integer, ſeries erit finita; ſin fractus, indefinita.

105. Combinando autèm terminos inter operandum inventos, breviùs exponere licet ſeriem fluxionalem, ponendo nimirùm, $\frac{m}{n} = p$; $\frac{m}{n} \cdot \frac{m}{n} - 1 = q$; $q \times \frac{m}{n} - 2 = r$; erit

$$paA^{\frac{m}{n} - 1}\text{, fluxio prima,}$$

$$qa^2A^{\frac{m}{n} - 2}\text{, fluxio ſecunda,}$$

$$ra^3A^{\frac{m}{n} - 3}\text{, fluxio tertia,}$$

$$sa^4A^{\frac{m}{n} - 4}\text{, fluxio quarta.}$$

Atque ulteriùs combinando, formam adhuc ſimpliciorem indicet ſeries; ponendo $pa = P$; $\overline{\frac{m}{n} - 1} \times aP = Q$; $\overline{\frac{m}{n} - 2} \times aQ = R$. $\overline{\frac{m}{n} - 3} \times aR = S$, &c. erit jam ſeries,

$PA^{\frac{m}{n} - 1} + QA^{\frac{m}{n} - 2} + RA^{\frac{m}{n} - 3} + SA^{\frac{m}{n} - 4} +$ &c; ipſi *ſeriei binomiæ*, in ſimplicitate non inferior.

De Theoremate Binomio.

106. In *Appendice* ad ANALYSIN ÆQUATIONUM, p. 33, &c. dudùm demonſtratum eſt, " *ex principiis Algebræ propriis,*" (ut ibi dictum,) ad mentem *Johannis Landen*, in tractatu ſuo *De Analyſi Reſiduâ*, theorema hoc longè celeberrimum, ſub variis ejus formulis, § 57; (quales protulit *Newtonus*, in Epiſtolis cum *Leibnitz* communicandis, an. 1676,) atque ſub formulâ uſitatiore, *Cor.* 1. qualem non ſemèl adhibuit ipſe *Newtonus*. Ex. gr. *Prop.* 45. *Lib.* 1. *Princip.* ubi ſtatuit *radium vectorem* $A^n = (T - X)^n = T^n - nXT^{n-1} + \frac{n^2 - n}{2}X^2 T^{n-2}$, &c. Atque iterùm, in *Scholio*, *Prop*, 93, *ordinatam* $(A + O)^{\frac{m}{n}} = A^{\frac{m}{n}} + \frac{m}{n}OA^{\frac{m-n}{n}} + \frac{mm - mn}{2nn}O^2A^{\frac{m-2n}{n}}$, &c.—Pleniorem et fuſiorem expli-

cationem Methodi *Landenianæ* inveniet lector horum ſtudioſus, apud *Maſeres*, in libro cui titulus *Scriptores Logarithmici*, Vol. II. p. 170, &c. et 176, &c.

107. Verùm hanc methodum perpendenti, atque iterùm ad examen revocanti, vix affirmare licet, ex principiis *merè Algebräicis* eam prorſùs peti: aſſumptio enim cui potiſſimùm innititur demonſtratio, nempè, "quòd *ſalvâ æquatione variari poteſt y ad libitum*, donec fiat æqualis ipſi *x* in ſingulis dimenſionibus fractionis $\frac{1+y}{1+x}$," calculum *fluxionalem* planè redolet, qualem in demonſtratione *tertii* caſûs, § 93, antecedentis jam perſpeximus, æquatis valoribus A, B, C, &c.

108. Admirandam ſanè intercedere analogiam inter *Theorema Binomium* et *Theoriam Fluxionum*, liquidò conſtat ex collatione terminorum ſeriei reſultantis, cum diverſis fluxionum ordinibus ſuprà expoſitis, § 105.

Exponat enim A quantitatem quamvis generantem, et *a* hujus *momentum*, ſeu fluxionem, ſive creſcendi ſive decreſcendi; deſignabit igitùr $A \pm a$ valorem *generantis*, et $(A \pm a)^{\frac{m}{n}}$ *genitæ*, ſimùl ac ſemèl fluentium. Hujus autem reſolutio dabit ſeriem, citiſſimè convergentem, cui apponitur ſeries quoque fluxionum diverſorum ordinum.

	Series Binomia.	*Series Fluxionum.*	
1.	$A^{\frac{m}{n}}$ - - - - -	$A^{\frac{m}{n}}$ - - - - -	*Fluens.*
2.	$\pm\frac{m}{n}aA^{\frac{m}{n}-1}$ - -	$\frac{m}{n}aA^{\frac{m}{n}-1}$ - - -	1 *Fluxio,*
3.	$\frac{m}{n}\cdot\frac{\frac{m}{n}-1}{2}\times a^2A^{\frac{m}{n}-2}$ -	$\frac{m}{n}\cdot\frac{m}{n}-1\;a^2A^{\frac{m}{n}-2}$ -	2 *Fluxio,*
4.	$\pm\frac{m}{n}\cdot\frac{\frac{m}{n}-1}{2}\cdot\frac{\frac{m}{n}-2}{3}\times a^3A^{\frac{m}{n}-3}$	$\frac{m}{n}\cdot\frac{m}{n}-1\cdot\frac{m}{n}-2a^3A^{\frac{m}{n}-3}$	3 *Fluxio.*
	&c.	&c.	

Hinc conſtat, quòd *primus* ſeriei terminus eſt *ipſa fluens*; *ſecundus* terminus, *ipſa prima* fluxio; reliqui autem terminis fluxionibus ſuperiorum ordinum, *proportionales* reſpectivè, et proinde pro his uſurpari poſſunt; per fluxionum definitionem, "menſuræ *quævis* rationum," &c.

Nam tertius terminus eſt ad ſecundam fluxionem in ratione $\frac{1}{2}$ ad 1, ſeu 1 ad 2.

Quartus terminus ad tertiam fluxionem in ratione $\frac{1}{2} \times \frac{1}{3}$ ad 1, ſeu 1 ad $2 \times 3 = 6$.

Quintus terminus ad quartam fluxionem in ratione $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$ ad 1, ſeu 1 ad $2 \times 3 \times 4 = 24$.

Et generalitèr, terminus quilibet $m + 1$, ad fluxionem ordinis m proximè inferioris, in ratione $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} - - - \frac{1}{m}$ ad 1, ſeu 1 ad $2 \times 3 \times 4 \times 5$ &c $\times\, m$; hoc eſt, in ratione datâ.

109. Et hinc facillimè quidèm, et optimè forsàn, ex theoriâ fluxionum *germanâ* derivatur conſtructio ſeriei ex reſolutione theorematis binomii reſultantis; nam quilibet ſeriei terminus $m + 1$ invenietur dividendo fluxionem ordinis m per $2 \times 3 \times 4 - - - m$. Sic terminus ſeriei *ſextus* eſt æqualis fluxioni *quintæ*, $t a^5 A^{\frac{m}{n} - 5}$, per 24×5 diviſæ. Et viciſsìm, ex ſerie binomiâ inveniri poſſunt fluxionum ordines.

110. Simplicitate atque facilitate longè præſtat hæc demonſtratio, ex fluxionum *ſerie* petita, *Landenianæ* iſti breviori, ex conſideratione *primæ* fluxionis tantùm hauſtæ; ab eâ quam dudùm expoſuit *Maclaurin*, § 748, vix diſcrepanti; quippe quæ primo *aſſumit*, quantitatem binomiam $(1+x)^{\frac{m}{n}}$ deſignari poſſe per ſeriem indicibus integris, perpetuò creſcentem, $1 + ax + bx^2 + cx^3 + dx^4$, &c. Et harum quantitatum fluxionibus utrinque inventis, erit $\frac{m}{n} \times (1+x)^{\frac{m}{n}-1} \times \dot{x} = a\dot{x} + 2bx\dot{x} + 3cx^2\dot{x} + 4dx^3\dot{x}$, &c. unde, dividendo omnes terminos per $\dot{x}$, emergit æquatio finalis $\frac{m}{n} \times (1+x)^{\frac{m}{n}-1} = a + 2bx + 3cx^2 + 4dx^4$, &c.; eadem prorsùs ad quam tanto circuitu tandem pervenit methodus *Landeniana*; vide *Analyſin Æquationum*, p. 38. Cujus iterùm reſolutio, calculo ſatìs operoſo, dabit valores ſingulorum co-efficientium, b, c, d, &c.

Hinc manifeſtò liquet, *Analyſin Reſiduam* ipſam, cujus laudes tantopere prædicat *Landen*, vix diſcrepare in principiis ſuis, licet longè operoſiorem ac tortuoſiorem, à *Methodo Fluxionum*; ac proinde pro larvâ ejus jure cenſeri poſſe.

111. Ex dictis conſtat, *ſeriem binomiam* $(A \pm a)^{\frac{m}{n}}$ citiùs convergere quam *ſeries fluxionum genitæ* $A^{\frac{m}{n}}$; ob decrementum terminorum prioris per denominatores ſucceſſivos terminis poſterioris ſubjiciendos, $1 \times 2 = 2$; $2 \times 3 = 6$; $6 \times 4 = 24$; &c.

Sic, $(A + a)^3 = A^3 + 3aA^2 + 3a^2A + a^3$.

Fluxiones, A^3, $= 3aA^2 + 6a^2A + 6a^3$.

De Methodo Inverſâ Fluxionum.

112. *Methodus Inverſa*, ſeu *Calculus Integralis*, viciſſim, *ex fluxionibus datis invenire fluentes* aggreditur. Eſtque "*hoc moleſtiſſimum et omnium difficillimum problema*," *Newtono* judice: nam hactenùs deſideratur methodus generalis eruendi fluentes directè, ſeu immediatè ex fluxionibus; ad particulares caſus tantùm reſtringitur inveſtigatio, eoſdémque, maximâ ex parte, curſum relegendo methodi directæ.

Auſpicari libet à caſibus ſimplicioribus.

113. *Caſ.* 1. *Fluxionum ſimplicium unius tantùm dimenſionis fluentes ſubſtitutione ſolâ inveniuntur.*

Sic fluxionis a, fluens eſt A; fluxionis $\frac{m}{n}a$, $\frac{m}{n}A$; vel fluxionis $\dot{x}$, fluens x; $\frac{m}{n}\dot{x}$, $\frac{m}{n}x$.

114. *Caſ.* 2. *Fluxionum ſimplicium plurium dimenſionum, fluentes reſolutione regularum methodi directæ inveniuntur.*

Sic quoniam fluentis A^3 fluxio fuit $3aA^2$; dividendo priorem per poſteriorem, prodit $\frac{A}{3a}$; quæ utique, multiplicando datam fluxionem, dabit fluentem viciſsìm.

ciſsìm. Itémque fluxionis $3\dot{x}x^5$, multiplicando per $\frac{x}{6\dot{x}}$, fluens erit $\frac{3x^6}{6} = \frac{1}{2}x^6$. Et generalitèr, formulæ $\frac{m}{n}\, a\mathrm{A}^{\frac{m}{n} - 1}$, multiplicando per $\frac{\mathrm{A}}{\frac{m}{n}a}$, eruetur fluens quæſita $\mathrm{A}^{\frac{m}{n}}$. Similitèr, formæ $a\mathrm{A}^{m-1}$ erit fluens $\frac{\mathrm{A}^m}{m}$, ut ex caſibus 3 et 5 methodi directæ conſtat.

115. Methodum directam excogitavit *Pacaſſi* inveniendi fluentes, inſtituendo analogiam: *Ex fluxione* ($\dot{x}$) *datâ, eliciatur nova quædam fluens* (y); *erítque hujus fluxio* ($\dot{y}$) *ad fluxionem datam* ($\dot{x}$) *ut fluens elicita* (y) *ad fluentem quæſitam* (x): et indè prodit $x = \frac{\dot{x}y}{\dot{y}}$.

Sit fluxio data $3a\mathrm{A}^2$; ſubſtituendo A pro a, emergit nova fluens $3\mathrm{A}^3$; inveniatur hujus fluxio, $9a\mathrm{A}^2$; undè prodibit $\frac{3a\mathrm{A}^2 \times 3\mathrm{A}^3}{9a\mathrm{A}^2} = \mathrm{A}^3$, fluens quæſita. Atque hæc regula, licet præcedente operoſior, in *fluentium correctione* per methodum inverſam inventarum, uſui eſſe poteſt.

116. *Fluxionis compoſitæ, unius tantùm dimenſionis formulæ* $\dot{x}y \pm x\dot{y}$, *invenietur fluens, ſubſtituendo fluentes pro fluxionibus, et dimidiando aggregatum*; nam $\frac{xy + xy}{2} = xy$. Et paritèr, formulæ $\frac{\dot{x}y - x\dot{y}}{y^2}$ fluens erit $\frac{xy + xy}{2y^2} = \frac{x}{y}$.

117. Veruntamen ſæpenumerò occurrunt quantitates fluxionales magìs compoſitæ, quarum fluentes per regulas prædictas neutiquàm eruere licet. Decurſu autem computorum ~~fluxionalium, complures *formulæ fluentiales*~~, formulis *fluxionalibus* magìs compoſitis reſpondentes, inventæ ſunt ab analyſtis celeberrimis, inter operandum; et potiſſimùm, à *Newtono*, *Cotes*, *Euler*, *Clairaut*, *[illegible]ley*, *Landen*, *Simpſon*, *Waring*, *De La Grange*, &c.; quarum præcipuæ ad calcem capitis *de Fluxionibus Logarithmorum* rejiciuntur, § 132.

De Correctione Fluentium.

---

118. Cæterùm fluentes hoc pacto repertæ, ſive per regulas præcedentes, ſive per formulas, haùd rarò correctione quâdam ulteriore egent, quo ad

amuſſim reſpondeant caſibus problematum fluxionalium particularibus inveniendis. Nam fluxionis $\dot{x}$ fluens eſſe poteſt vel x, vel $x \pm c$, ſeu x conſtanti quâdam quantitate, c, aucta vel diminuta; adeò ut fluens jam inventa, conditionibus quibuſdam, quæ problema aliquatenùs reſtringunt, poterit forsàn non accuratè reſpondere. Unde neceſſe erit ulteriùs fluentem veram eruere ex ipſius problematis conditionibus, relationem fluentium ad invicèm in certo quodam temporis puncto, aut mutationis ſtatu, definientis ſeu limitantis. Reducatur igitùr generalis æquatio fluentialis ad iſtum ſtatum particularem, ſubſtituendo pro fluentibus ipſis cognitos, quos ſortiuntur, valores. Deinde ſubducatur æquatio reſultans ex æquatione generali, et reſidua utrinque æquationem correctam fluentium ex regulis inventarum, elicient.

Ut, ſi æquatio fluxionalis ſit $\dot{z} = ax^3 \dot{x}$, æquatio fluentialis poteſt eſſe vel $z = ax^4$, vel $z = ax^4 \pm c$. Si ſimùl evaneſcunt x et z, ſubſtituendo o pro x et z, fiet æquatio $0 = 0 + c$. Adeóque $c = 0$: quo in caſu, ax^4 veram fluentem exponet, evaneſcente c.

2. At, ſi, evaneſcente z, reſtat $x = b$, finitæ cuidam quantitati cognitæ; fiet æquatio fluentialis, in iſto caſu, $0 = ab^4 + c$: unde prodit $c = -ab^4$; et ſubſtituto hoc valore, prodit $z = ax^4 - ab^4$.

3. Fiat jam $z = d$, iſto in ſtatu ubi x fit æqualis b; et his valoribus ſubſtitutis, emerget æquatio fluentialis $d = ab^4 + c$; adeóque erit $c = d - ab^4$; et, ſubſtituto iterùm hoc valore pro c, prodit $z = ax^4 - ab^4 + d$, valor nimirùm correctus fluentis quæſitæ.

4. Idem quoque breviùs expedire licet, ſimùl et ſemèl ſubſtituendo valores quoſvis contemporaneos quantitatum x et z. Nam ſi ab æquatione fluentiali $z = ax^4$, ſubducatur novus valor $d = ab^4$, fiet reſiduum $z - d = ax^4 - ab^4$; adeóque $z = ax^4 - ab^4 + d$, ut anteà: aut igitùr evaneſcentibus b et d, aut æquatis $+d$ et $-ab^4$, fiet ipſa fluens quæſita ax^4.

De Fluxionibus Diverſas Fluentes admittentibus.

119. Nullâ re magìs arguitur imperfectio *Methodi Inverſæ*, quam quòd eadem æquatio fluxionalis ſeu differentialis, aliquando duas ſolutiones, omninò ab invicèm diverſas, ſortiatur; quod primò detexit *Clairaut*, *Mem. Acad.* 1734, et poſteà confirmarunt *Euler*, *D'Alembert*, *De La Place*, *De La Grange*, et *Le Gendre*. Vide M. R. Append. Vol. 28, p. 537.

Hujuſmodi eſt æquatio differentialis $x\,dy\,dx - dy^2 = y\,dx^2 - dy\,dx$, ſive ſecundùm notationem fluxionalem, $x\dot{x}\dot{y} - \dot{y}^2 = y\dot{x}^2 - \dot{y}\dot{x}$. Huic enim ſolvendæ æquè reſpondent vel æquatio fluentialis $4y = x^2 + 2x + 1$, vel $2ax - 2x = -4y + 1 - a^2$.

Similitèr quoque æquationem differentialem $a\,dy^2 + x\,dy^2 - y\,dy\,dx = x\,dy\,dx - y\,dx^2$ æquè ſolvent, vel $\frac{x}{\sqrt{y}} - \sqrt{y} = \sqrt{4a}$, vel $b^2y - 2bx + 2by = -4a$.

Supereſt ut brevitèr proponere aggrediar applicationem Fluxionum ad *Logarithmotechniam*, planiùs ut ſpero, et pleniùs, quam hactenùs; ob ingentem Logarithmorum utilitatem in Calculis Trigonometricis, &c. abbreviandis; ídque libentiùs, quòd inter celeberrimorum *Scriptorum Logarithmicorum* congeſta opera locum habere permittatur hæc *Analyſis Fluxionum*, mole quidem parva, ſed quæ tamèn, pro rerum difficultate, ſubtilitate, varietate, et pondere, (quæ fortaſſe fuſiùs explicari debuiſſent,) quantumvis impar, tanto honore non prorsùs indigna videatur.

De Fluxionibus Logarithmorum.

120. *Logarithmi*, quaſi λογων ἀριθμοι, " *Rationum numeri exponentes*" ſunt *numeri arithmeticè proportionales*, *numeris geometricè proportionalibus reſpondentes*, ſeu ſecundùm definitionem vulgatam, " *Numerorum proportionalium comites æquidifferentes*:" ut in ſeriebus inſequentibus Logarithmorum vulgarium:

Numeri	&c.	10^3,	10^2,	10^1,	10^0,	10^{-1},	10^{-2},	10^{-3},	&c.
					vel				
		1000,	100,	10,	1,	·1,	·01,	·001.	
Logarithmi		3,	2,	1,	0,	− 1,	− 2,	− 3.	

De utilitate inſigni Logarithmorum in omni genere Computorum Arithmeticorum, Trigonometricorum, abbreviando operationes *Multiplicationis*, *Diviſionis*, *Involutionis* et *Evolutionis*, hoc loco monuiſſe tantùm ſufficiat: adeat harum rerum ſtudioſus librum anteà memoratum, cui titulus *Scriptores Logarithmici*, à viro cl. *Maſeres* congeſtum et editum.

121. In omni ſyſtemate Logarithmorum, ſtatuitur logarithmus *unitatis* eſſe nihil, aut 0. In omni ſyſtemate igitùr, *ſumma logarithmorum cujuſvis numeri logiſtici, ejúſque reciproci*, cum ſint æquales et contrariis ſignis affecti, fiet 0, ſeu *evaneſcet*.

In Syſtemate Logiſtico vulgari, ſeu *Briggiano*, ſumitur *unitas* pro logarithmo numeri 10; at in Syſtemate Hyperbolico, ſeu *Neperiano*, *unitas* prodit logarithmus numeri 2·7182818, &c.

122. In quovis autem ſyſtemate, numerus qui creſcit vel decreſcit eâdem ratione ac logarithmus ejus, fit quaſi *modulus* iſtius ſyſtematis: erítque *modulus unius ſyſtematis ad modulum alterius, ut logarithmus dati numeri in uno ad logarithmum ejuſdem numeri in altero.*

Sic in ſyſtemate vulgari, logarithmus numeri 10 eſt 1; at in hyperbolico, logarithmus 10 eſt 2·3025850, &c. In hyperbolico, item modulus eſt 1; undè prodit modulus ſyſtematis vulgaris $= \frac{1{\cdot}0000000}{2{\cdot}3025850} = 0{\cdot}4342944$.

Erítque *ratio modularis*, in ſyſtemate hyperbolico 1 ad 1, ſive ratio æqualitatis; at in vulgari, 0,4342944 ad 1, nempè ratio data.

123. *Fluxiones logarithmorum*, fluxionibus *ordinis ſecundi* aſcribendæ ſunt; cum ipſi logarithmi *fluxionibus primis* reſpondent: ut ex definitionibus conſtat.

Methodus háſce inveniendi propoſitionibus inſequentibus fundatur.

---

124. *Prop.* 1. *Quantitatum logiſticarum fluxiones ſunt ipſis quantitatibus proportionales.*

Nam quoniam quantitates ipſæ proportione *geometricâ* creſcunt vel decreſcunt, erunt differentiæ earum ipſis quantitatibus proportionales; per converſam *Lem.* 1. *Lib.* II. PRINCIP. Et proinde, fluxiones, quæ variantur ut hæ differentiæ, erunt quoque ipſis quantitatibus proportionales. Q. E. D.

125. *Prop.* 2. *Fluxio quantitatis logiſticæ eſt ad fluxionem logarithmi ejus, ut ipſa quantitas ad modulum ſyſtematis logiſtici.*

Per propoſitionem præcedentem eſt n, fluxio quantitatis N, ad m, fluxionem quantitatis M, ut N ad M. Deſignet M *modulum*, et quoniam m in hoc caſu eſt conſtans, ſubſtitui poteſt pro fluxione logarithmi N quoque, cum hujus logarithmus eſt in datâ quâdam ratione ad modulum. Erit igitùr n, fluxio ipſius N ad l, fluxionem logarithmi ejus, ut N ad M. Q. E. D

126. *Prop.* 3. *In quovis ſyſtemate logarithmorum, fluxio logarithmi cujuſvis dati æquatur fluxioni ipſius quantitatis logiſticæ, in modulum ductæ, et per ipſam quantitatem diviſæ.*

Nam (per *præced.*) eſt $n : l :: N : M$; et invertendo, $l : n :: M : N$, ſive $l = \frac{nM}{N}$. Q. E. D.

127. *Prop.* 4. *In ſyſtemate hyperbolico, fluxio logarithmi cujuſvis dati æquatur fluxioni ipſius quantitatis logiſticæ, per ipſam quantitatem diviſæ.*

Nam in hoc ſyſtemate, $M = 1$, et proinde, $l = \frac{n}{N}$. Q. E. D.

Idem quoque *geometricè* illuſtrare licet ex naturâ arearum *hyperbolæ rectangulæ*, inter curvam, aſymptotas et ordinatas intercluſarum. Deſignantibus enim x aſymptoti abſciſſam; y, ordinatam; et m, quadratum conſtans, contentum utique aſymptotis et parallelis à vertice ductis, ſeu modulum; (per naturam hyperbolæ,) erit $xy = m$; adeóque, $y = \frac{m}{x}$; et multiplicando utrinque per $\dot{x}$, erit $\dot{x}y = \frac{\dot{x}m}{x}$, ſive ob $m = 1$, $= \frac{\dot{x}}{x}$; ſed $\dot{x}y$ eſt fluxio logarithmi dati; (per naturam hyperbolæ et logarithmorum communem). Ergo, &c. Q. E. D.

Hinc manifeſtum eſt quòd *hyperbola rectangula* cujus axis eſt aſymptotos, eſt curva logiſtica, ſeu curva ad deſignandum logarithmos, et eos quaſi ante oculos ponendum, accommodata: creſcentibus enim abſciſſis in progreſſione *Arithmeticâ*, decreſcent ordinatæ in progreſſione *Geometricâ*.

De Fluxionibus Quantitatum Exponentialium.

---

128. *Quantitates exponentiales* ſunt aliarum quantitatum poteſtates quarum indices ſunt variabiles; ſicut vel e^x, vel y^x; quarum prima eſt poteſtas quantitatis conſtantis e variabili indice x deſignata, altera verò eſt poteſtas quantitatis variabilis y, variabili indice x paritèr deſignata.

Caſ. 1. Fiat $e^x = z$. Eríque $\log. z = \log. e, \times x$; adeóque per præced. $\frac{\dot{z}}{z} = \log. e, \times \dot{x}$; et proinde, $\dot{z} = \log. e \times \dot{x}z, = \log. e \times \dot{x}e^x$. Sive *fluxio quantitatis exponentialis* (e^x) *æquatur ipſi quantitati* (e^x), *in indicis fluxionem* ($\dot{x}$), *et lateris logarithmum* (log. e) *continuè ductæ.* Q. E. D.

Caſ. 2. Fiat $y^x = z$. Eríque $\log. z = \log. y \times x$; adeóque $\frac{\dot{z}}{z} = \log. y \times \dot{x} = \frac{\dot{y}}{y} \times \dot{x}$; et proinde, $\dot{z} = \frac{\dot{y}}{y} \times \dot{x}z = \frac{\dot{y}}{y} \times \dot{x}y^x = \dot{y} \times \dot{x}y^{x-1}$. Sive *fluxio quantitatis exponentialis variabilis* (y^x) *æquatur fluxioni lateris* ($\dot{y}$) *in fluxionem indicis* ($\dot{x}$) *et co-efficienti* (y^{x-1}) *continuè ductæ.* Q. E. D.

N. B. In

N. B. In demonſtratione hujus caſûs, ex *Hutton's Mathematical Dictionary* excerptâ, occurrit *erratum* preli haud contemnendum, $+$ pro $=$, bis. Art. *Fluxions*, p. 489.

De Logarithmis Imaginariis.

129. Logarithmi quantitatum impoſſibilium, ut $\sqrt{-a^2} = a\sqrt{-1}$, appellantur *Imaginarii*; quales ſunt fluentes fluxionum quarundam impoſſibilium, ſicut $\frac{\dot{x}}{x\sqrt{-1}}$. Nam quoniam fluxionis $\frac{\dot{x}}{x}$ fluens eſt logarithmus *verus* quantitatis realis x; pari ratione, erit fluxionis $\frac{\dot{x}}{x\sqrt{-1}}$ fluens, quoque logarithmus *imaginarius* quantitatis impoſſibilis $x\sqrt{-1}$.

Veruntamen ubi tales quantitates in problematum ſolutionibus occurrunt, facilè tranſmutari poſſunt in *arcus circulares*, aut *ſectores veros*, ex *hyperbolicis impoſſibilibus*. Vide MACLAURIN, *Of the Analogy betwixt Circular Arcs and Logarithms*; *Fluxions*, § 762:—PLAYFAIR, *On the Arithmetic of Impoſſible Quantities*; *Phil. Tranſ.* an. 1778, p. 318; or, *Analyſ. Æquat.* § 223, et *Scriptores Logarithmicos*, tomum ſecundum, in paginis 381, 382, &c - - - ad 585, ubi methodus quam invenit celeberrimus Leibnitzius anno 1676 pro extenſione duarum regularum à Cardano olìm traditarum de radicibus æquationum cubicarum formæ $x^3 + qx = r$ in omnibus earum caſibus, et formæ $x^3 - qx = r$ in primo earum caſu, ſeu cum $\frac{rr}{4}$ eſt major $\frac{q^3}{27}$, ad ſecundum caſum formæ ſecundæ, in quo $\frac{rr}{4}$ eſt minor ipsâ $\frac{q^3}{27}$ (qui *caſus irreducibilis* à ſcriptoribus Algebræ appellatur,) fusè et accuratè explicatur.

130. Analogiam inter *Methodum Fluxionum* et *Logarithmorum*, ex recentioribus primus detexiſſe videtur ſagaciſſimus *Kepler*; qui logarithmos ritè definivit "*menſuras rationum*," ſive "*numeros ratiuncularum inter rationem numeri alicujus ad unitatem intercedentium.*" Cæterùm antiquis penitùs latuiſſe hanc analogiam, haud facilè perſuadear, vel ex ſolo opere celeberrimo *Apollonii*, *De Sectione Rationis*; quâcum ad amuſſim quadrat hæc expoſitio *Kepleriana*; atque, non injuriâ, *Apollonio* forsàn debuit editor ejus, ſagaciſſimus *Halley*, methodum ſuam egregiam, ſed paulò reconditiorem exquirendi et conſtruendi logarithmos, ſive "*numeros rationum exponentes*," (optimâ nominis definitione uſus), per varias combinationes et *ſectiones rationum*, ſeries logiſticas citiſſimè convergentes inſtituendo; qui omnium primus, ut fertur, *problema inverſum* extudit: *dato logarithmo invenire numerum ejus logiſticum.*

131. Methodum *Halleianam*, celeberrimam at perdifficilem quidèm, acutiùs expoſuit *Jones*, in *Synopſi Palmariorum Matheſeos*; pleniſſimè autem et luculentiſſimè perveſtigávit analyſta ex peritiſſimis *Maſeres*, in magno ſuo repoſitorio *Scriptorum Logarithmicorum*, tomo primo in paginis 235, 236, &c - - - 383, et tomo ſecundo in paginis 76, 77, &c - - - - 152.

132. Accedit ultimò TABULA præcipuarum FORMULARUM FLUXIONALIUM et FLUENTIALIUM hactenùs repertarum, ex Cl. *Hutton's Mathematical Dictionary* excerpta.

Formulæ Fluxionum atque Fluentium.

	Fluxionum.	*Fluentium.*
I.	$x^{n-1}\dot{x}$	$\frac{1}{n}x^{n}$.
II.	$(a \pm x^{n})^{m-1}x^{n-1}\dot{x}$	$\pm\frac{1}{mn}(a \pm x^{n})^{m}$.
III.	$\frac{x^{mn-1}\dot{x}}{(a \pm x^{n})^{m+1}}$.	$\frac{1}{mna} \times \frac{x^{mn}}{(a \pm x^{n})^{m}}$.
IV.	$\frac{(a \pm x^{n})^{m-1}\dot{x}}{x^{mn+1}}$	$\frac{-1}{mna} \times \frac{(a \pm x^{n})^{m}}{x^{mn}}$.
V.	$mx^{m-1}y^{n}\dot{x} + ny^{n-1}x^{m}\dot{y}$. vel $(m\dot{x}y + nx\dot{y})x^{m-1}y^{n-1}$. vel $(\frac{m\dot{x}}{x} + \frac{n\dot{y}}{y})x^{m}y^{n}$.	$x^{m}y^{n}$.
VI.	$mx^{m-1}y^{n}z^{r}\dot{x} + nx^{m}y^{n-1}z^{r}\dot{y} + rx^{m}y^{n}z^{r-1}\dot{z}$ vel $(m\dot{x}yz + nx\dot{y}z + rxy\dot{z})x^{m-1}y^{n-1}z^{r-1}$ vel $(\frac{m\dot{x}}{x} + \frac{n\dot{y}}{y} + \frac{r\dot{z}}{z})x^{m}y^{n}z^{r}$.	$x^{m}y^{n}z^{r}$.

	Fluxionum.	*Fluentium.*
VII.	$\frac{\dot{x}}{x}$	Log. x.
VIII.	$\frac{x^{n-1}\dot{x}}{a \pm x^n}$	$\pm\frac{1}{n}$ log. of $a \pm x^n$.
IX.	$\frac{x^{-1}\dot{x}}{a \pm x^n}$	$\frac{1}{na}$ log. of $\frac{x^n}{a \pm x^n}$.
X.	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{a-x^n}$	$\frac{1}{n\sqrt{a}}$ log. of $\frac{\sqrt{a}+\sqrt{x}^n}{\sqrt{a}-\sqrt{x}^n}$.
XI.	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{a-x^n}$	$\frac{2}{n\sqrt{a}}$ × arc. tang. $\frac{\sqrt{x^n}}{a}$, vel $\frac{1}{n\sqrt{a}}$ × arc. coſin. $\frac{a-x^n}{a+x^n}$.
XII.	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{(\pm a+x^n)^{\frac{1}{2}}}$	$\frac{2}{n}$ log. $\sqrt{x^n}+\sqrt{\pm a+x^n}$.
XIII.	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{(a-x^n)^{\frac{1}{2}}}$	$\frac{2}{n}$ × arc. ſin. $\frac{\sqrt{x^n}}{a}$, vel $\frac{1}{n}$ × arc. verſ. ſin. $\frac{2x^n}{a}$.
XIV.	$\frac{x^{-1}\dot{x}}{(a \pm x^n)^{\frac{1}{2}}}$	$\frac{1}{n\sqrt{a}}$ log. $\frac{\pm\sqrt{a\mp x^n}\pm\sqrt{a}}{\sqrt{a\pm x^n}+\sqrt{a}}$.
XV.	$\frac{x^{-1}\dot{x}}{(-a+x^n)^{\frac{1}{2}}}$	$\frac{2}{n\sqrt{a}}$ × arc. ſec. $\sqrt{\frac{x^n}{a}}$, vel $\frac{1}{n\sqrt{a}}$ × arc. coſin. $\frac{2a-x^n}{x^n}$.

	Fluxionum.	*Fluentium.*
XVI.	$\overline{dx - x^2}^{\frac{1}{2}} \times \dot{x}$	$\frac{1}{2}$ circ. ſegm. ad diam. d et verſ. ſin. x.
XVII.	$c^{nx}\dot{x}$	$\frac{c^{nx}}{n \log. c}$.
XVIII.	$xy^{x} \log. y + xy^{x-1}\dot{y}$	y.
XIX.	$\overline{a + x^n}^{m} x^{rn-1}\dot{x}$	$\frac{\overline{a + x^n}^{m+1} x^{rn-n}}{ns} \times \left(\frac{1}{1} - \frac{r-1 \,.\, a}{s-1 \,.\, x^n} + \frac{r-1 \,.\, r-2 \,.\, a^2}{s-1 \,.\, s-2 \,.\, x^{2n}}, \text{\&c.}\right)$

De Formularum Applicatione.

133. Methodùs applicationis hujuſmodi eſt.

Conferatur fluxio propoſita cum formulis; nempè, partes correſpondentes cum correſpondentibus fluxionis tabularis quâcum conſentit; et hiſce valoribus ſubſtitutis in formulam fluentis generalem, invenietur fluens quæſita.

Ex. gr. 1. *Invenire fluentem fluxionis* $3x^{\frac{5}{3}}\dot{x}$. Conſentit hæc cum *primâ* formulâ. Adeóque, propter $n - 1 = \frac{5}{3}$, erit $n = \frac{5}{3} + 1 = \frac{8}{3}$; erítque $\frac{1}{n}x^n = \frac{3}{8}x^{\frac{8}{3}}$. Et proinde fluens quæſita, multiplicando per 3, co-efficientem fluxionis datæ, erit $3 \times \frac{3}{8}x^{\frac{8}{3}} = \frac{9}{8}x^{\frac{8}{3}}$.

2. *Invenire fluentem fluxionis* $\frac{x^2\dot{x}}{1+x^3}$. Conſentit hæc cum *octavâ* formulâ. Nam prodit $\frac{x^{-1}\dot{x}}{a+x^n} = \frac{x^2\dot{x}}{1+x^3}$, itémque $n = 3$, et $a = 1$; quibus ſubſtitutis in formulam fluentis generalem, prodit fluens quæſita $\frac{1}{3}$ log. $\overline{1+x^3}$.

3. *Invenire fluentem quantitatis* $5\dot{x}x^2\sqrt{c^3 - x^3}$ ſeu $5\dot{x}x^2(c^3 - x^3)^{\frac{1}{2}}$. Quadrat hæc cum formulâ *ſecundâ*. Erit igitur $a = x^3$; $m - 1 = \frac{1}{2}$, ſeu $m = \frac{3}{2}$, et $n = 3$; quibus ſubſtitutis, prodit fluens quæſita, $5 \times \frac{1}{\frac{3}{2} \times 3}(c^3 - x^3)^{\frac{3}{2}}$ $= \frac{10}{9}(c^3 - x^3)^{\frac{3}{2}}$.

134. Ubi quantitas fluxionalis propoſita nullis ex his formulis finitis reſpondeat, inveniri poteſt ejus fluens quàm proximè, *reſolvendo propoſitam in ſeriem infinitam; vel per diviſionem, vel per theorema binomium; et inveniendo ſingulorum terminorum fluentes; quorum ſumma ad libitum continuatorum, dabit fluentem quæſitam, proximè aut quam proximè.*

4. Invenire fluentem $\frac{1 - x}{1 + x - x^2}\dot{x}$. Dividendo numeratorem per denominatorem, prodit $\dot{x} - 2x\dot{x} + 3x^2\dot{x} - 5x^3\dot{x} + 8x^4\dot{x}$, &c. Et inventis ſingulorum fluentibus, prodit ſumma $x - x^2 + x^3 - \frac{5}{3}x^4 + \frac{8}{5}x^5$, &c. fluens quæſita.

De correctione fluentium per regulas aut per formulas inventarum, vide ſuprà, § 118.

BRITISH MUSEUM 29 FEB 68

APPEN-

APPENDIX I.

DE ANALYSI ANTIQUÂ.

Antiquam exquirite Matrem. VIRG.

APPENDIX I.

DE ANALYSI ANTIQUÂ.

§ 135. ANALYSIN Antiquam, qualem exercebant clariſſimi Geometræ veteres, *Euclides*, *Archimedes*, *Apollonius*, &c. jamdudùm laudavimus; optimam ſanè, judice in hîſce verſatiſſimo, *Halley*, ANALYS. ÆQUAT. § 128. Et quàm curioſâ felicitate eandem excoluit et auxit *Newtonus*, ex operibus ejus philoſophicis abundè conſtat. Placet igitùr, hoc tempore, ubi labefactantur PHILOSOPHIÆ NATURALIS vel ipſa PRINCIPIA MATHEMATICA, et juventuti noſtræ horum ſtudioſæ potiùs commendantur, (utpotè majoris evidentiæ, et rigoris geometrici,) "*functiones quædam analyticæ*," "*fluxiones hypotheticæ*," è *Scholâ Gallicâ* novatrice profluentes, eximiam illam *Præfationem Pappi Alexandrini*, ex Editione Halleianâ *Apollonii de Sectione Rationis*, hoc loco edere ab integro.

136. "Locus *de Reſolutione* inſcriptus, *Hermodore* fili, ut paucis dicam, propria quædam eſt materia in eorum uſum deſignata qui, perceptis *communibus elementis*, in GEOMETRIA facultatem ſibi deſiderant *inveſtigandi ſolutiones problematum*, et in hunc finem ſolummodò utilis. Traditur autem à tribus viris, *Euclide*, nempe, Elementorum ſcriptore, *Apollonio Pergæo*, et *Ariſtæo* ſeniore.

Quid ſit *Analyſis*, **ſive** *Reſolutio.*

1. "Procedit verò, per *Methodum Reſolutionis et Compoſitionis*. *Reſolutio* (αναλυσις) autem eſt *methodus quæ à quæſito quaſi jam conceſſo, per ea quæ deinde conſequuntur ad concluſionem aliquam cujus ope compoſitio fit*, (επι τι ὁμολογȣμενον εν συνθεσει) *perducamur*. In reſolutione enim quòd quæritur *ut jam factum* ſupponentes, ex quo antecedente hoc conſequatur, expendimus; iterúmque quodnam fuerit hujus antecedens; atque ita deinceps; uſque, dum in hunc modum regredientes, in *aliquid jam cognitum*, locóque *principii* habitum, incidamus. Atque hic proceſſus *Analyſis* vocatur, quaſi dicas (αναπαλιν λυσιν) *Inverſa Solutio*.

"E con-

Quid ſit *Syntheſis*, ſive *Compoſitio.*

"E contrario autem, in *compoſitione* (εν συνθεσει,) cognitum illud in *reſolutione* ultimo loco acquiſitum, ut *jam factum* præmittentes, et quæ ibi conſequentia erant, hìc ut antecedentia, naturali ordine diſponentes, atque inter ſe conferentes, tandem ad *conſtructionem* quæſiti pervenimus. Hoc autem vocamus *Syntheſin.*

"Duplex autem eſt *analyſeos* genus: vel enim eſt veri indagatrix, dicitúrque *Theoretica*, vel propoſiti inveſtigatrix, ac *Problematica* vocatur. In *theoretico* autem genere, *quod quæritur, reverà ita ſe habere ſupponentes*; *ac deinde per ea quæ conſequuntur, quaſi vera ſint*, (ut ſunt *ex hypotheſi*,) *argumentantes*, *ad evidentem aliquam concluſionem* (επι τι ὁμολογȣμενον) *procedimus.* Jam ſi *concluſio* illa *vera* ſit, *vera* eſt quoque *propoſitio* de quâ quæritur; ac demonſtratio reciprocè reſpondet *analyſi:* ſi verò in *falſam* concluſionem incidamus, *falſum* quoque erit de quo quæritur.

"In *problematico* verò genere, *quod proponitur, ut jam cognitum ſiſtentes, per ea quæ exinde conſequuntur tanquam vera, perducimur ad concluſionem aliquam.* Quòd ſi concluſio illa *poſſibilis* ſit ac ϖοριςικη (quod mathematici *datum* appellant) *poſſibile* quoque erit quod proponitur: Et hìc quoque demonſtratio reciprocè reſpondet *analyſi:* ſi verò incidamus in concluſionem *impoſſibilem*, erit etiàm problema *impoſſibile.*

"*Dioriſmus* autem (διορισμος), ſive *determinatio*, eſt quâ diſcernitur *quibus* conditionibus, et quot modis, problema effici poteſt. Atque hæc de *reſolutione* et *compoſitione* dicta ſunto."——

137.—"Verùm perpendendum eſt (notat *Halley*,) aliud eſſe problema *aliqualitèr* reſolutum dare, quod modis variis plerúmque fieri poteſt; aliud, *methodo elegantiſſimâ* id efficere: *Analyſi* breviſſimâ et ſimùl perſpicuâ; *Syntheſi* concinnâ et minimè operoſâ. Hoc veteres præſtitiſſe, argumento eſt *Apollonii* Liber *De Sectione Rationis.*"

138.—"Ex his, credo, manifeſtum eſt (monet quoque *Newtonus*, *De Reſolutione Quæſtionum Geometricarum*,) quid ſibi velint *Geometræ*, cum jubent, '*putes factum eſſe quod quæris:*' Nullo enim inter cognitas et incognitas quantitates habito diſcrimine, *quaſlibet* ad ineundum calculum *aſſumere* potes, quaſi *omnes* ex præviâ ſolutione eſſent notæ; et non ampliùs de *ſolutione problematis* ſed de *probatione ſolutionis* ageretur."—"Convenit ut fingas quæſtionem de *ejuſmodi datis et quæſitis* propoſitam eſſe per quas arbitreris te poſſe ad æquationem *facillimè* pervenire."—"Cum varios ordines quibus termini quæſtionis evolvi poſſint perſpexeris, *E ſyntheticis quoſlibet adhibe* aſſumendo lineas tanquam *datas*, à quibus ad *alias facillimus videtur progreſſus*, et *ad ipſas viciſsìm difficillimus.*"—"Manuducit *analyſis* ad *compoſitionem:* ſed compoſitio non priùs verè fit, quam liberatur ab omni analyſi. Inſit compoſitioni vel minimum analyſeos, et compoſitionem veram nondùm aſſecutus es. *Compoſitio* in ſe perfecta eſt, et à mixturâ *ſpeculationum analyticarum* abhorret."

Hæc *Newtonus*, in "LIBRO" illo verè aureo, "DE COMPOSITIONE ET RESOLUTIONE," quem concinnavit, ut firmior "*ſtaret ſuper vias antiquas*" Juventus *Britannica*; et quo meliùs obviàm iret pravæ iſti novitati ex Scholâ *Carteſianâ* graſſanti, quâ *Carteſius*, "mixturis ſpeculationum *analyticarum*" nimìs indulgendo, evidentiam et certitudinem *Geometriæ*, primus, obſcurare et labefactare cœpit; et proinde, quaſi per antitheſin operi *Carteſiano*, cui nomen erat, per abuſum, "GEOMETRIA:" Librum ſuum, *Newtonus*, titulo magìs idoneo, ARITHMETICÆ UNIVERSALIS inſcripſit.—Atque ad hunc librum, mole parvum, at pondere et ſubtilitate materiæ eximium ("cui forsàn ſecundum vix inveniatur, niſi opus analyticum *Algebra Clairautiana*") interpretandum atque illuſtrandum, jamdudùm edebam ANALYS. ÆQUATIONUM, anno 1784. Vide *Præfat.* illius operis.

139. Methodum *Analyticam* et *Syntheticam* non ſolùm in *Matheſi*, ſed etiàm in *Mechanicâ Rationali*, optimè exponit *Newtonus*, ſub finem *Opticorum*.

"As in *Mathematics*, ſo in *Natural Philoſophy*, the inveſtigation of *difficult* things by the Method of *Analyſis*, ought ever to precede the Method of *Compoſition*.

"*This Analyſis* conſiſts in making *experiments* and *obſervations*, and in drawing *general concluſions* from them by *Induction*; and admitting of no *objections* againſt the concluſions, but ſuch as are taken from *experiments* or other *certain truths*: for *hypotheſes* are not to be regarded in EXPERIMENTAL PHILOSOPHY. And although the arguing from experiments and obſervations be *no demonſtration* of general concluſions, yet it is the *beſt way* of arguing which the nature of things admits of, and may be looked-upon as ſo much the ſtronger, by how much the induction is more general: and if *no exception* occur from phenomena, the concluſion may be pronounced generally. But if, at any time afterwards, any exception ſhall occur from *experiments*, it may then begin to be pronounced with *ſuch exceptions as occur*."

"By this way of *Analyſis*, we may proceed from *Compounds* to *Ingredients*, and from *Motions* to the *Forces* producing them; and, in general, from *Effects* to their *Cauſes*; and from *particular* cauſes to *more general* ones, till the argument end in the moſt general:—this is the Method of *Analyſis*."

"And the *Syntheſis* conſiſts in *aſſuming* the *Cauſes* diſcovered and eſtabliſhed, as *Principles*; and by them *explaining* the *phænomena* proceeding from them, and *proving* the explanations."

And in the Preface to his PRINCIPIA, he gives the following elegant ſummary of this ſtatement:—

“ *Omnis* enim Philosophiæ *difficultas* in eo verſari videtur, ut à *phænomenis motuum* inveſtigemus *Vires Naturæ* ; deinde, ab *his viribus*, demonſtremus *phænomena reliqua.*”

140. And he furniſhes the following inſtances of his application of this method, in thoſe two immortal works, his Optics and Principia :

“ In the two firſt books of theſe Optics, I proceeded, by *this Analyſis*, to diſcover and prove the *original differences* of the Rays of Light, in reſpect of *Refrangibility*, *Reflexibility*, and *Colour*, &c.—And theſe diſcoveries being *proved*, may be aſſumed, in the Method of *Compoſition*, for *explaining the phænomena* ariſing from them : an inſtance of which I gave in the end of the firſt book, [in the ſolution of the colours and figure of the *Rainbow*, from the refraction and reflexion of the Sun's rays in drops of falling rain.]

“ In libro autem tertio [Principiorum] exemplum hujus rei propoſuimus per explicationem Systematis Mundani : Ibi enim, ex *phænomenis cœleſtibus*, per propoſitiones in libris prioribus *mathematicè* demonſtratas, derivantur *vires gravitatis*, quibus corpora ad Solem et Planetas ſingulos tendunt : deinde, ex *his viribus*, per propoſitiones etiàm *Mathematicas*, deducuntur motus *Planetarum*, *Cometarum*, *Lunæ* et *Maris*.”

141. And he expreſsly ſtates that he adopted this method from the Ancients :—

“ Cum Veteres *Mechanicam* [*Rationalem*, quæ per demonſtrationes *accuratè* procedit,] (uti auctor eſt *Pappus*,) in *rerum naturalium* inveſtigatione maximi fecerint ; et Recentiores, miſſis *formis ſubſtantialibus*, et *qualitatibus occultis*, *phænomena* Naturæ ad *leges mathematicas* revocare aggreſſi ſint : *viſum eſt in hoc tractatu* Mathesin *excolere, quatenùs ea ad* Philosophiam *ſpectat*.”

How highly Newton admired the *Ancient Geometricians*, we learn from Dr. Pemberton :—

“ He often cenſured the handling of *Geometrical* ſubjects by *Algebräick* calculations ; and his Book of Algebra he called by the name of *Arithmetica Univerſalis*, in oppoſition to the injudicious title of *Geometry*, which Des Cartes had given to the Treatiſe in which he ſhews how the Geometrician may aſſiſt his invention by ſuch kind of computations. He frequently praiſed Slusius, Barrow, and Huygens for not being influenced by the *falſe taſte* which then began to prevail : he uſed to recommend the laudable attempt of Hugo d'Omerique to reſtore the *Ancient Analyſis* ; and very much eſteemed Apollonius's book *De Sectione Rationis*, for giving us a clearer notion of that Analyſis than we had before. Dr. Barrow may be eſteemed as having ſhewn a compaſs of *invention* equal, if not ſuperiour, to any of the Moderns, our Author only excepted : but

NEWTON particularly recommended HUYGENS's ſtyle and manner; he thought him *the moſt elegant of any mathematical writer of modern times, and the trueſt imitator of the Ancients.* Of *their* taſte and mode of demonſtration our Author always profeſſed himſelf a great admirer; and even cenſured himſelf for not following them yet more cloſely than he did: and ſpoke with regret of his miſtake, at the beginning of his Mathematical Studies, in applying himſelf to DES CARTES and other *Algebräick* writers, before he had conſidered *the Elements* of EUCLID with that attention which ſo excellent a writer deſerves."

Abſolutis hactenùs VINDICIIS NEWTONIANIS, quatenùs *Elementa Metaphyſica* et *Principia Mathematica* reſpiciunt, (quibus fundantur opera ejus eximia Analytica et Philoſophica) adversùs objectiones *Gallorum*, *Germanorum*, *Anglorúmque*, horum evidentiam et certitudinem, aut malignè, aut imperitè, aut oſcitantèr, inſectantium; ſupereſt *altera* et *acrior cura*, ut repellatur crimen multo gravius, "*facinúſque majoris abollæ*," quaſi PHILOSOPHIÆ atque THEOLOGIÆ cultor ille, omnium modeſtiſſimus et ſaniſſimus, fuiſſet et ipſe fautor dogmatis praviſſimi atque turpiſſimi quod *Materialiſmum* vocant, Naturam *Mentis Humanæ* merè mechanicam ſtatuentis, et "*divinæ particulam auræ*," (etiàm ab antiquiſſimis agnitæ,) extinguere geſtientis; ipſum NUMEN SUMMUM—OPTIMUM, SAPIENTISSIMUM, MAXIMUM et POTENTISSIMUM omnium, quaſi *Animam Mundi*, *Ætheream*, pro nefas! abſurdè et impiè fingentis, et deliramenta "*inſanientis ſapientiæ*," iterùm recoquentis, per Philoſophiam pravam et Mythologiam *Leibnitzianam*, graviſſimóſque errores inde prognatos; hoc *ſæculo rationis* (ut dicitur) infauſto, *Revelationem* aſpernante.

BRITISH MUSEUM 29 FEB 68

APPENDIX II.

DE ÆTHERE VIBRATORIO,—DE MODO SENTIENDI,

ET

DE ENTE SUPREMO.

"AUCTORIBUS UTI OPTIMIS *in omnibus cauſis, et debet et ſolet valere plurimum: et primum quidem,* OMNI ANTIQUITATE; *quæ, quo propriùs aberat ab ortu et divinâ progenie, hoc meliùs ea, fortaſſe, quæ erant vera, cernebat.*" CIC.

Ουκ εγνω ὁ Κοσμος, δια της σοφιας, ΤΟΝ ΘΕΟΝ.

"*Non novit Mundus, Sapientiæ [humanæ] ope,* DEUM." PAUL.

APPENDIX II.

DE ÆTHERE VIBRATORIO.

"*CE grand homme* (NEWTON) *voyoit à travers d'un voile, ce qu'un autre ne diſtingue qu'à peine avec un Microſcope!*" DAN. BERNOULLI *.

142. Nuper, cum perlegerim opus eximium, perutile, tempeſtivum, à viris bonis ubique laudatum,—"*Proofs of a Conſpiracy againſt all the Religions and Governments of Europe*, &c. by *John Robiſon*, A. M. *Profeſſor of Natural Philoſophy*, &c. *Edinburgh*,"—incidi in cenſuram haud levem *Patris Philoſophiæ Britannicæ*, p. 483.

"Were it poſſible for the departed ſoul of *Newton* to feel pain, he would ſurely recollect with regret that *unhappy hour*, when, provoked by Dr. *Hooke*'s charge of plagiariſm, he firſt threw out his *whim* of a *Vibrating Æther*; to ſhew what might be made of an *Hypotheſis*: (for Sir *Iſaac Newton* muſt be allowed to have *paved the way* for much of the *Atomical Philoſophy* of the Moderns.) Newton's *Æther* is aſſumed as a *fac totum* by every *precipitate ſcioliſt*, who, in deſpite of logic, and in contradiction to all principles of mechanics, gives us *theories* of *muſcular motion*, of *animal ſenſation*, and even of *intelligence* and *volition*, by the *undulations of ætherial fluids*.

"Not one of a hundred of theſe *theoriſts* can go through the fundamental theorem of all this doctrine, (the 47th *Prop.* of the *Second Book of the* PRINCIPIA); and not one of a thouſand know that *Newton*'s inveſtigation is *inconcluſive*:—yet they talk of the effects and modifications of thoſe undulations as familiarly as if they could demonſtrate the propoſitions in *Euclid*'s *Elements*."

Talis viri ictus, PHYSICAM excolentis, et NEWTONUM quaſi præceptorem adamare profitentis, acerbiùs lædunt: Antiſtiti ipſi ſcientiarum venerando, annon exclamare fas ſit, jamdudùm hujuſmodi crimina auſcultanti :—" Si id *Cinerem* aut *Manes* credis curare ſepultos (*a*)."

---

* Vide Note (*e*) poſthàc.

5

—" *Pol, me occidiſtis* Amici !—
Ohe jam ſatis eſt !—Et Tu, Brute ! ! !—

Telum imbelle jacis ; imò, tuum in cerebrum facilè retorquendum, vibranti medium tranſverberans ictu."

143. Pluſquàm viginti retrò annis, Diſquiſitionem edebam, anno 1778, *Dublin, De Sonis* et *De Modificationibus Atmoſphæræ*, ſecundùm Theoriam *Newtonianam*,—primitias Studiorum meorum Philoſophicorum,—dum, apud Academicos *Dublinienſes*, *Socius* degerem. Et, cum iſta diſquiſitio forsàn nondum ad manus pervenerit Profeſſoris *Robiſon*, me operam vel illi viro clariſſimo déque Republicâ optimè merenti, haùd ingratam impenſurum ſpero, ſi exinde pauca excerpſerim, hanc rem ſpectantia.

Primo igitùr in limine, monendus eſt, *Newtonum*, haudquaquam " *opinionum* " ſeu *hypotheſeon* vanarum ſtudioſum, nec novum neque inauditum " *commentum* " de " *æthere* quodam *vibratorio* " finxiſſe ; ſed vetuſtum omninò, atque à philoſophis ſanioribus receptum : § 42.

Zeno, audiendi facultatem ità exponit, *Laert. Lib.* 7. § 158.—Ακουειν δε, του μεταξυ του τε φωνουντος και του ακουοντος, αερος πληττομενου σφαιροειδως, ειτα κυματουμενου, και ταις ακοαις προσπιπτοντος· ὡς κυματουται το εν τῃ δεξαμενῃ ὕδωρ, κατα κυκλους, ὑπο του εμβληθεντος λιθου.

" *Auditus* fit, ùbi aer inter loquentem audientémque interpoſitus, pulſatur *ſphæricè* ; deinde undulatur, auribúſque impingit : ſicuti undulatur *aqua* in ciſternâ, *ſecundùm circulos*, injecto lapide."

(*a*) Quantum à diſputationibus controverſiiſque abhorruit mite *Newtoni* ingenium, conſtabit ex Epiſtolâ ejus *ad Oldenburgh*, Oct. 24, 1676.

—" Five years ago, (1671,) when, at the perſuaſion of my friends, I had reſolved to publiſh a Treatiſe of the *Refraction of Light and Colours*, which I had then by me ; I began again to turn my thoughts to theſe *Serieſes* [for determining the Areas of Curves] ; and wrote likewiſe a Treatiſe upon them, that I might publiſh both together.—But having wrote you a letter upon the *Reflecting Teleſcope*, in which I explained briefly my notions on the *Nature of Light*, ſomething unexpected fell out, which made me imagine it concerned me to write to you ſpeedily about printing that Letter. Immediately upon the back of this, I met with *frequent interruptions* from ſeveral perſons' letters, taken up with *objections*, and *other* things, which *frighted* me entirely from executing my deſign ; and made me find fault with myſelf for my *imprudence*—that, by catching at *a ſhadow*, I had been ſo far *deprived of the peace and quiet of my own mind* ; *which is a thing of more ſubſtantial worth*."

At laſt *Newton* found out an expedient that rid him of much of this troubleſome and teazing correſpondence : when objections were ſtated againſt the accuracy or validity of his *Optical* experiments, he ſimply replied—" I *doubt the fact* ;"—" *repeat the experiment*,"—inſtead of entering into any diſcuſſion or vindication about his own.

Diogenis atque *Platonis* placita quoque, ſic refert *Plutarchus*, *De Placit. Philoſ.* Lib. 4. cap. 16.—Διογενης, του εν τῃ κεφαλῃ αέρος ὑπο της φωνης τυπτομενου και κινουμενου· Πλατων, και ὁι απ᾽ αυτου, πληττεσθαι τον εν τῃ κεφαλῃ αέρα, τουτον δε ανακλασθαι εις τα ἡγεμονικα, και γινεσθαι της ακοης την αισθησιν.

“ *Diogeni* [placet hoc effici] aëre intrà caput à voce pulſato atque commoto. *Platoni*, ejúſque ſequacibus, aërem intrà caput pulſari, deinde ad partes *imperatorias* reflecti; atque hoc pacto ſenſum auditûs fieri.”

Lucretius quoque phænomena *Lucis* et *Sonorum* diſertè confert: *Lib.* 4.

“ Quod ſupereſt, non eſt mirandum quâ ratione,
Quæ loca per, nequeunt oculi res cernere apertas;
Hæc loca per, *voces* veniant *auréiſque laceſſant.*
Conloquimur clauſis foribus, quod ſæpe videmus:
Nimirùm, quia *vox per flexa foramina* rerum
Incolumis tranſire poteſt; *ſimulacra* renutant;
Perſcinduntur enìm, niſi recta foramina tranant,
Qualia ſunt vitri, ſpecies quâ travolat omnis.
  Præterea, *partes in cunctas dividitur vox*;
Ex aliis aliæ quoniam gignuntur, ubi una
Diſſiluit ſemèl in multas exorta: (quaſi ignis
Sæpe ſolet ſcintilla ſuos ſe ſpargere in ignes):
Ergò *replentur loca vocibus, abdita retrò,*
Omnia quæ circùm fuerint, ſonitúque cientur:
At *ſimulacra* viis directis omnia tendunt,
Ut ſunt miſſa ſemèl; quapropter *cernere* nemo
Se ſuprà potis eſt, at *voces* accipere extrà.”

Atqui hìc habes quaſi ſeminarium totius *Doctrinæ Sonorum*, rationalis et experimentalis, in difficillimâ parte difficillimi operis, nempè, in *Sectione octavâ, Lib. 2. Princip.* “ *De Motu per fluida propagato* expoſitæ et demonſtratæ.”

144. Haudquaquàm patitur hujus inſtituti ratio, de Doctrinâ ſubtiliſſimâ Pulſuum et Vibrationum Aëris, ſeu *Medii Ætherei*, jam diſſerere, quæ vel mathematicè doctos non parùm morantur; accuratiùs autem exponitur fuſiúſque illuſtratur tota hæc doctrina, in Diſquiſitione prædictâ *De Sonis*.

Unam ſaltèm obſervationem inde depromere non pigebit, p. 139, *de veritate propoſitionis illius fundamentalis* 47, à Profeſſore *Robiſon* denegatæ:

“ *Le Sœur* et *Jacquier* veritatem propoſitionis deducunt ex vibrationibus partium *chordæ elaſticæ*, quæ peraguntur viribus in ratione diſtantiarum à medio vibrationum loco: ſed ſingulæ aëris particulæ motum chordæ *mutuantur* ſucceſſivè: ergò, inſtar chordæ ipſius, *moventur* viribus, &c.—At hæret deductio niſi oſtendere poſſent, *omnia corpora* ſonitum edentia *legem* chordæ elaſticæ in tremoribus ſuis ſervare, [*aërémque* ſimiliter agitari].

“ *Rowning, Nat. Phil. Part* 2. p. 48, veritatem propoſitionis immediatè deducit ex *lege repulſivâ partium aëris:*—Nam particula quævis, preſſione partium à tergo, versùs particulas antecedentes propelletur viribus quæ ſunt ut intervalla centrorum reciprocè: atque ideò, ut diſtantiæ hujus particulæ à primo ſuo loco directè.”

Ex hâc lege *vis elaſticæ* ſeu *repulſivæ*, (quæ à *Newtono* mathematicè demonſtratur, *Prop.* 23. *Lib.* 2. *Princip.* et experimentis confirmatur, § 4, *De Sonis*), elicitur concluſio, Quod *ſingulæ fluidi elaſtici tremuli particulæ, motu reciproco breviſſimo euntes et redeuntes, accelerantur ſemper et retardantur in ratione diſtantiarum à medio vibrationum loco:* hoc eſt, *pro lege penduli in cyclo-eïde oſcillantis.*

145. Demonſtratio *Newtoniana*, quâ inſtituitur analogia hæc elegantiſſima inter *vibrationes aëris* et *oſcillationes penduli*, longè ſubtiliſſima, eſt prorſus analytica, ex ipſis fontibus ANALYSIS ANTIQUÆ hauſta:

“ *Fingamus medium* [*elaſticum*] *tali motu* [*vibratorio*] *à cauſâ quâcunque cieri; et videamus quid indè conſequatur,* &c.”

Propoſitionis autèm 47 ipſa concluſio, “*Aërem, pulſibus per medium elaſticum propagatis, cieri ſeu agitari pro lege oſcillantis penduli,*” abundè confirmatur experimentis de variis ſonorum generibus, tormentorum diſploſionibus, &c. factis ad inveſtigandam *velocitatem ſonorum*, (ſeu rationem ſecundùm quam rectâ progrediuntur), in ITALIA, obſervantibus Academicis *Florentinis*; in ANGLIA, obſervantibus *Newtono, Derhamo,* &c.; in AMERICA AUSTRALI, obſervantibus Academicis *Gallis* atque *Hiſpanis*; et nupèr etiàm in AFRICA, propè oſtium *Nili*, victoriâ illâ inſigni navali apud Sinum *Aboukir*, Aug. 1, 1798, à claſſe Anglicâ ſub imperio ducis magnanimi Horatii Nelſon, de claſſe Reipublicæ Gallicæ reportatâ; ubi fragor navis *L'Orient*, exploſæ, *Roſet'am* pertingebat, juſto poſt fulgorem viſum intervallo temporis, ratione habitâ diſtantiæ et fluxûs uniformis ſonorum (*b*).

(*b*) Si obſervationibus M. *Pouſſielgue*, *Gallorúmque* adſtantium, tempore illo trepidationis bellicæ iniquo et infauſto, fides eſt adhibenda, fragor navis *L'Orient*, in ſinu *Aboukeriano* exploſione combuſtæ, ſtationem ſuam apud *Roſettam* pertingebat tempore *duorum minutorum primorum* poſt fulgorem viſum. At progagatur ſonus mediâ velocitate 1142 pedum circitèr, tempore unius minuti ſecundi, et proindè 13 milliarium Anglicorum tempore unius minuti primi. Diſtabant igitùr *Galli* 26 milliaria Anglica à loco exploſionis, *ad minimum*, recto curſu.

Augenda eſt enim hæc diſtantia, ob *calores æſtivos* iſtius regionis, in ratione 1142, ſeu potiùs pedum Gallicorum 1070 ad 1101; ſicùt pedum Gallicorum 1101, ſingulis minutis ſecundis, determinabatur velocitas ſonorum ad inſulam *Cayenne*; (intervallo 20230 hixapedarum *(toiſes)*, ſeu milliarium Anglicorum 25 ferè, geometricè menſurato à *Condamine* Academicíſque Gallis; atque adeò accuratè ut, ex quinque experimentis factis per tormentorum exploſiones, quatuor vix uno ſemi-ſecundo temporis ab invicèm diſcrepârunt. Diſtantia igitùr, hâc ratione aucta, correctior emerget milliarium 27 ferè. Atque iterùm forsàn augenda eſt, ob ingentem fragorem navis exploſæ, intenſiſſimos tremores, eoſdémque forsàn principio latiſſimos, excitantis. Academicorum *Anglorum* certè æquum foret rem determinare, geometricè menſurando diſtantiam ſtationis apud *Roſettam*, à nave *L'Orient*; ad perficiendam aut corrigendam Theoriam *Newtonianam*, experimento adeò illuſtri, minimè negligendo.

Conſenſus igitùr theoriæ cum experimentis fidelitèr inſtitutis ubicunque regionum ſatìs confirmat *Analyſin Newtonianam* ritè inſtitutam fuiſſe; nam *veritas concluſionis infert veritatem præmiſſorum.*—Vide *De Sonis*, § 47; ubi datur Tabula Obſervationum *Florentinorum*, &c.; necnon *Append.* p. 71; ubi demonſtrationes harum propoſitionum valdè difficillium, 47 et 49, pleniùs exhibentur, atque notis illuſtrantur.

146. Nec eſt cur de *Æthere Vibratorio* cavillationes excitentur:—vox *æther* à Newtono uſurpatur pro *aëre puriſſimo, vaporibus* ſordibúſque heterogeneis defæcato:—"hi enim, (cum ſint alterius elateris et alterius toni,) vix, et ne vix quidèm, participant motum *aëris veri* quo ſoni propagantur." *Princip.* p. 373.

Hunc *ætherem*, ſeu *medium æthereum*, ingenti elatere (ſeu elaſticitate) eſſe præditum, docet *Newtonus, Optics*, p. 325, his verbis:

"That the *elaſtic force* of this medium is exceeding great, may be gathered from the ſwiftneſs of it's vibrations; *ſounds* move about 1140 Engliſh feet in a ſecond of time, and in 7 or 8 minutes of time they move about 100 Engliſh miles: *Light* moves from the Sun to us in about 7 or 8 minutes of time; which diſtance (ſuppoſing the Sun's horizontal parallax to be about 12″,) is about 70 millions of Engliſh miles; and by conſequence, above 700,000 times ſwifter than ſounds: and therefore the elaſtic force of this medium muſt be above 490,000 million of times greater than the elaſtic force of the air [at the earth's ſurface]"—in proportion to their reſpective rarities (*c*).

147. The aſtoniſhing *rarefaction* of which the air is capable is remarked, *Optics*, p. 341, from experiment and from computation:

"Mr. *Boyle* has ſhewn that air may be rarefied about 10,000 times in veſſels of glaſs; and the heavens are much emptier of air than any *vacuum* we can make below. For, ſince the air is compreſſed by the weight of the incumbent atmoſphere, and the denſity of the air is proportional (or it's rarity reciprocally proportional) to the force compreſſing it, it follows, by computation, [*Princip. Lib.* 2. *Prop.* 22. *Cor.* et *De Sonis*, § 14.] that, at the height of about 7½ Engliſh miles from the earth, the air is 4 times rarer than at the ſurface of the earth; and at the heights of 22½, 30, or 38 miles, it is reſpectively 64, 256, or 1024 times rarer, or thereabouts; and at the height of 76, 152, 228 miles, it

---

(*c*) *Newton, Principia*, p. 405, eſtimates the Sun's horizontal parallax, lower, at 10½."—And the calculations made ſince the Tranſit of Venus in 1761, reduce it further to 8″,6; which will enlarge the Sun's diſtance from us to upwards of 95 millions of miles; and conſequently raiſe the proportion of the elaſtic force of the ætherial medium to upwards of 900,000 million times greater than that of our air.

The calculation depends on the velocities of the pulſes (or propagation of the vibrations) of elaſtic fluids being in a compound ſubduplicate ratio of their elaſticities and rarities: *Princip.* II. 49. *Cor.* 2.

is about a million, a billion, or a trillion, times rarer; and ſo on:" inſomuch that, in the *Principia*, p. 512, *Newton* computes, that "a globe of our air one inch in diameter, at the elevation of one ſemidiameter of the earth (about 4000 miles), would, by it's expanſive power, fill all the planetary regions, far beyond the orbit of *Saturn*."

148. Non temère igitùr conjicit *Newtonus*, *Princip.* p. 530—"De *ſpiritu quodam ſubtiliſſimo corpora craſſa pervadente, et in iiſdem latente*; cujus vi et actionibus particulæ corporum ad minimas diſtantias ſe mutuò *attrahant*; et contiguæ factæ, *cohærent*; et corpora *electrica* agunt ad diſtantias majores, tàm repellendo quàm attrahendo corpuſcula vicina; et *lux* emittitur, reflectitur, refringitur, inflectitur et corpora calefacit; et *ſenſatio omnis* excitatur, et *membra animalium ad voluntatem moventur: vibrationibus ſcilicèt hujus ſpiritûs, per ſolida nervorum capillamenta ab externis ſenſuum organis ad cerebrum, et à cerebro in muſculos, propagatis*."

149. Fuſiùs hæc exponit, *Optical Quæries*:

"Is not *animal motion* performed by the power of this [*ætherial*] *medium*, excited in the *brain* by *the power of the will*, and propagated from thence through the ſolid, pellucid, and uniform, *capillamenta* [or minute fibres] of the nerves into the muſcles for contracting and dilating them?"—Vide § 195.

[N. B.] "I ſuppoſe that the *capillamenta* of the nerves are each of them *ſolid* and *uniform*, that *the vibrating motion of the ætherial medium* may be propagated along them from one end to the other uniformly and without interruption (for obſtructions in the nerves create *palſies*): and that they may be ſufficiently uniform, I ſuppoſe them to be *pellucid* when viewed *ſingly*, though the reflexions in their cylindrical ſurfaces may make the *whole nerve*, compoſed of many capillamenta, appear opaque and white."

Cautiſſimè igitùr diſtinguit *Newtonus* his locis, inter *mechanicas* operationes fluidi hujúſce ætherei ſeu "*ſpiritûs ſubtiliſſimi*"—et *mentem* ipſam ſeu *voluntatem*, quæ háſce ſentit, et quodam modo nobis prorsùs incognito, dirigit.

150. Et quàm ſollicitè diſtinguit quoque inter modificationes corporum, et ſenſationes quas in mente excitant! in *Optics*, p. 108.

"If at any time I ſpeak of *light* and *rays* as coloured or endued with colours, I would be underſtood to ſpeak not philoſophically and properly, but groſsly, and according to ſuch conceptions as vulgar people, in ſeeing all theſe experiments, would be apt to frame. For the rays, to ſpeak properly, are not coloured: in them there is nothing elſe but *a certain power and diſpoſition to ſtir up a ſenſation of this or that colour*: for as SOUND in a bell or muſical ſtring, or other *ſounding body*, is nothing but a *trembling motion*; and in the *air* is nothing

but that motion *propagated* from the object; and in the *ſenſorium* 'tis a *ſenſe* of that motion under the form of *ſound:* ſo COLOURS in the *object* are nothing but a *diſpoſition* to *reflect* this or that ſort of rays more copiouſly than the reſt; in the *rays*, they are nothing but their *diſpoſitions to propagate* this or that motion into the *ſenſorium*; and in the *ſenſorium*, they are ſenſations of thoſe motions under the forms of *colours.*"

DE SENSORIO ET DE VI SENTIENDI.

151. QUANTAS tragedias excitavit hæc vox technica *ſenſorium!* Reclamant *Leibnitziani:* "*Newtonum* hìc habes reum confitentem, et vim ſentiendi omnem ex cauſis *mechanicis* ſolventem!"

Sed ipſum audiamus, de induſtriâ definientem hanc vocem, "*the place of ſenſation.*"

"Is not the *ſenſory* of animals that *place* to which the ſenſitive ſubſtance is preſent, and into which the ſenſible ſpecies of things are carried through the nerves and brain; that there they might be *perceived* by their immediate preſence to *that ſubſtance?*"

And this diſtinction between the *organs of ſenſation* and the *ſenſitive ſubſtance* itſelf was long ago noticed by *Socrates*, in the *Phædo* of *Plato*, thus ridiculing the *Mechanical Philoſophiſts* of his day:

"It is true indeed, that without *bones* and *nerves*, and ſuch other *organs* as I poſſeſs, it would be impoſſible to *act* as I thought fit; but to aſſert that I do what I do by their *agency*, and not from *choice* of the beſt, would be downright abſurdity, and a mode of argument truly ridiculous."—Thus nobly ſtating his decided choice, rather to remain, and ſubmit to the unjuſt ſentence of the *Athenian* Judges, than to ſave his life by eſcaping to *Megara,* and ſeeking an aſylum among the *Bœotians.*

But was this preference merely the reſult of *vibratiunculæ* of the brain?

De induſtriâ item monet *Newtonus*, ne perperàm intelligatur *conjectura* ſua ſagaciſſima de modo ſentiendi;—ſe nullatenùs *theoriam* proponere: *Princip. fine.*

"Sed hæc *paucis* exponi non poſſunt; neque adeſt *ſufficiens copia experimentorum* quibus *leges* actionum hujus *ſpiritûs* [*ætherei*] accuratè determinari et monſtrari debent."

152. Conjecturæ *Newtonianæ* de actionibus hujuſce *ſpiritûs*, ſeu *fluidi electrici*, adſtipulari fas ſit, recitando phænomena quædam, quæ mihi ipſi obtigerunt abhinc circitèr decennio; necnon viro clariſſimo *Maclaurin*, moribundo; quæ ſatìs confirmare videntur *exiſtentiam talis ſpiritûs in ipſis cerebri penetralibus actù latentis, et functionibus animi rectoris peragendis apprimè forsàn neceſſarii*; intimo quodam et incomprehenſibili nexu inter *volitiones* et *actiones* ſubſiſtente, mediantibus organis corporeis. Nam, ſecundùm *Locke*:

> " It may ſeem probable, that the *conſtitution* of the body does *ſometimes* influence *the memory*; ſince we *ſometimes* find a *diſeaſe* quite ſtrip the mind of all it's *ideas*, and the flames of a *fever* in a few days calcine all thoſe *images* to duſt and confuſion, which ſeemed to be as laſting, as if graved in marble."—*Eſſay*, *Book* II. c. 10.—See alſo § 195.

153. Æſtate anni 1789, ipſe in *febrem bilioſam* incidi, ex diutinis laboribus, aſſiduis ſtudiis, nimiíſque curis contractam. Et quùm febriens inſomnio miſerè laborarem, medicus quidam ex peritioribus, mihíque amicus, *laudani* liquidi potionem adminiſtrabat, ad reducendum ſomnum. At ſpem prorsùs fefellit: licèt enim potio bìs térve repeteretur, manebat inſomnium; et, quod notatu digniſſimum eſt, unaquâque vice, decèm circitèr minutis temporis poſt potionem hauſtam, phænomenon ſingulare occurrebat: *quaſi ſcintillæ ignis affatìm effluxerunt ex oculis meis, velutì ictu percitis*. Me ipſo igitùr deprecante, intermiſſa eſt potio.—Exinde autèm valetudine ægerrimâ afflictus ſum; affectionibus nervoſis, et capitis lento dolore potiſſimùm, per triennium; obſtructis quasì viribus animi;—inutilis mihi, inutilis propinquis et amicis, inutilis reipublicæ, ετωσιον αχθος αρȣρης.

Tandèm in plenitudine temporis atque mœroris, ubi cernere erat indicia haùd obſcura *paralyſeos* jamjàm imminentis, opitulatus eſt MEDICUS SUMMUS—'Η γαρ ΔΥΝΑΜΙΣ αυτȣ εν ΑΣΘΕΝΕΙᾼ τελειȣται.—Mihi enìm ſuggeſſit ILLE DEUS, ut ex deſidiâ ferè continuâ me extricarem, et ambulando ſtrenuè me exercerem ad ſudorem uſque, ſi quâ ratione cieretur defluxio à cerebro, quod onere quaſi incumbente valdè erat oppreſſum; et longè ultrà expectationem fuit ſucceſſus: nam in fine hebdomadis unius tali exercitio occupatæ, ſex aut ſeptem horis per diem, defluxio humoris lymphatici, ſeu aquei, ſubitò erupit, cum admodùm incaleſcerem ambulando, et quaſi à vertice ipſo onus incumbens ſublatum eſt, humore per glandes maxillares abundè ſtillante in os; et in fine hebdomadis alterius, gradatìm exſiccatâ defluxione, ad perfectam integrâmque valetudinem reſtitutus ſum; et meliorem quidèm quam per viginti antè annos. Diffugêre febres; redii ad me-ipſum, redii ad ſtudia diù intermiſſa, redditus ſum propinquis amicíſque, redditus patriæ—Iterùm mihi conceſſa eſt, per miſericordiam divinam, *Mens ſana in corpore ſano*.

DEO OPTIMO MAXIMO LAUS! *Luc.* xvii. 18.

154. Recolebam febriens iſto tempore, hujuſmodi phænomenon, vel eidem valdè ſimile, viro clariſſimo *Maclaurin* obtigiſſe moribundo; et inde erat quod averſatus atque deprecatus ſum, quaſi infauſtum et malè ominatum foret, potionem illam *laudani*, quæ id manifeſtò excitaverat; vaporibus forsàn laudani, (ut poſteà ſuſpicatus ſum), à ſtomacho calido diſſolutis, (qui, utpote valdè bilioſus, actionem laudani tunc reſpuebat,) et inde aſcendentibus in cerebrum; et *ignem electricum* ibi latentem expellentibus per nervos opticos; vaporibus ipſis in aquam poſteà converſis.

Caſus *Maclaurini* ità deſcribitur à biographo optimo:

—"His behaviour, during this tedious and painful diſtemper—[a *dropſy of the belly*, for which he had been thrice tapped]—was ſuch as became a PHILOSOPHER and a CHRISTIAN, calm, chearful and reſigned: his *ſenſes* and *judgment* remaining in their full vigour, till within a few hours of his death. Then, for the firſt time, his *Amanuenſis*, to whom he was dictating the laſt Chapter of the following work, [An Account of Sir *Iſaac Newton*'s Philoſophical Diſcoveries,] (in which he proves the wiſdom, the power, goodneſs, and other attributes of the DEITY,) obſerved ſome heſitation or repetition: no *pulſe* could then be felt in any part of his body; and his hands and feet were already cold. Notwithſtanding this extremely weak condition, he ſate in his chair, and ſpoke to his friend Dr. *Munro*, with his uſual ſerenity and ſtrength of reaſon; deſiring the Doctor to account for a *phænomenon* which he then obſerved in himſelf; *flaſhes of fire ſeemed to dart from his eyes; while, in the mean time, his ſight was failing, ſo that he ſcarce could diſtinguiſh one object from another.*—In a little time after this converſation, he deſired to be laid upon his bed; where, on *Saturday* the 14th of *June*, 1746, aged 48 years and four months, he had an *eaſy paſſage* (*) *from this world to that ſtate of bliſs*, which he had the moſt elevated ideas of, and which he moſt ardently longed to poſſeſs."

"*Let* ME *die the death of* THE RIGHTEOUS;
And let MY *laſt end be like* HIS!!"

(*) —Μεταϐεϐηκεν εκ τȣ θανατȣ εις την ζωην. *John*, v. 24.

"*Migravit* ex morte in vitam."

"Neque enim aſſentior iis qui hæc *nuper* diſſerere cœperunt," [*Lucretius*, &c.] "*Cum corporibus animos ſimùl interire, et omnia morte dèleri.*" *Cicero de Amicitiâ.*

"M. Video te *altè ſpectare* et *velle in cœlum migrare.*
A. *Spero fore ut contingat id nobis.*" *Cicero, Tuſcul.* I.

Maclaurin's Laſt Words.

155. The concluding, unfiniſhed paragraph of that ſublime Diſcourſe is too curious in the *Hiſtory of Man*, too precious to the *Chriſtian Philoſopher*, to be withheld on this occaſion :—

—" As MAN is undoubtedly the *chief being* upon this globe ; and *this globe* may be no leſs conſiderable in the moſt valuable reſpects than any other in the *ſolar* ſyſtem ; and *this ſyſtem*, for aught we know, not inferior to any in the UNIVERSAL SYSTEM : So, if we ſuppoſe *Man* to *periſh*, without ever arriving at a more competent knowledge of NATURE, than the *very imperfect* one he attains in his *preſent* ſtate, by *analogy* (*d*) (or parity of reaſon), we might conclude, that the *like deſires* would be fruſtrated in the *inhabitants* of all the other *planets* and *ſyſtems* ; and that the *beautiful ſcheme* of NATURE would never be unfolded, but in an exceedingly imperfect manner, to *any* of them :—This, therefore, naturally leads us to conſider our *preſent* ſtate,—as only *the dawn* or *beginning of our exiſtence* ; and as a ſtate of *preparation* or *probation*, for farther advancement : which appears to have been the opinion of the *moſt judicious Philoſophers* of old. [See *Heb*. xi. 13—16. and Note (*e*).]

" And

(*d*) It was a maxim, recorded of *Hermes*, and aſcribed to the primitive *Chaldæan* or *Perſian Magi* (the eſtabliſhed prieſthood of the Babyloniſh and Perſian Empire)—

Συμπαθη ειναι τα ανω τοις κατω.

" *That what paſſes in the heavens above is analogous to what paſſes on earth below.*"

But this ſage maxim, like many others, derived from *patriarchal* wiſdom, in proceſs of time was much abuſed by ſucceeding corrupt *poets* and degenerate *philoſophers*, receding from the native purity and ſimplicity of Revelation, and impiouſly attributing *human* paſſions to *celeſtial* beings. See § 169.

(*e*) The following curious and valuable anecdote we owe to Profeſſor *Robiſon*, p. 236 :—

" *Daniel Bernoulli*, the moſt elegant mathematician, the only philoſopher and the moſt worthy man of that celebrated family, ſaid to a gentleman (Dr. *Staehling*), who repeated it to me, that *when* reading ſome of thoſe *wonderful gueſſes* of Sir *Iſaac Newton*, the ſubſequent demonſtration of which has been the chief ſource of fame to his moſt celebrated commentators,—his *mind* has ſometimes been ſo overpowered by *thrilling emotions*, that he has wiſhed that moment to be his laſt ; and that it was *this* which gave him *the cleareſt conception of the happineſs of heaven*."

And ſhould not theſe " *thrilling emotions*," to which, I am perſuaded, Profeſſor *Robiſon* is no ſtranger, induce him to treat the Doctrine of a " *Vibrating Æther* " with more reſpect, ſo widely removed from " a *whim* made out of an *hypotheſis*," by *Newton*'s ingenuity ?—Should he not rather rank it among the moſt profound of his *wonderful gueſſes*,—approaching very nearly to the dignity of a *theory*, founded on ſuch *experiments* as the preceeding ?—How otherwiſe, are we to account for ſuch *thrilling emotions* of exſtatic delight ? ſuch *convulſive horrors* of agonizing fear ? ſuch ſtrange effects, as the change of the colour of the hair of a condemned criminal *from black* to *white* in the courſe of a ſingle night ? ſuch fatal and deadly diſruptions of the *material* vehicle, as are produced by ſudden

“ And whoever attentively conſiders THE CONSTITUTION OF HUMAM NATURE, particularly the *deſires* and *paſſions* of men, (which are *greatly ſuperior* to their preſent objects,) will eaſily be perſuaded, that MAN was *deſigned* for *higher* views than thoſe of this life: theſe, the AUTHOR OF NATURE *may* have in reſerve; to be opened-up to us at *proper periods* of time, and after *due preparation:* ſurely it is in HIS power to grant us a much greater *improvement* of the faculties we already poſſeſs; or even to endow us with *new* faculties (of which at this time we have no idea) for penetrating farther into the SCHEME OF NATURE, and approaching nearer to HIMSELF—THE FIRST AND SUPREME CAUSE:—We know not how far it was proper or neceſſary, that WE ſhould not be let into knowledge *at once*, but ſhould advance *gradually*; that, by comparing new objects or new diſcoveries with what was known to us before, our improvement might be more compleat and regular: or how far it may be neceſſary or advantageous that INTELLECTUAL BEINGS ſhould paſs through a kind of *infancy of knowledge*; for new knowledge does not conſiſt ſo much in our having acceſs to a new object, as in comparing it with others already known; obſerving it's *relations* to them (or diſcerning what it has in

---

and overpowering tranſports of joy or grief?—Theſe are notorious *effects:* and how they are to be accounted for, upon attentive conſideration of the *conſtitution of human nature*, compoſed of a *mind* operating upon it's receptacle or *ſenſorium*, otherwiſe than by ſome inconceivable *ſtimulus* of the nervous ſyſtem, conveyed through *muſcular vibrations*, or ſome *impulſe* analogous thereto, I leave to his ingenuity to ſuggeſt.

2. To the taſte and piety of Profeſſor *Robiſon* (whom I hail as a *Brother*, notwithſtanding our diſagreement in one inſtance,) we are alſo indebted for circulating in his deſervedly popular work, *Maſon*'s Sacred Ode, drawn from the pureſt ſources of the *Pierian Spring:*

“ Think not THE MUSE whoſe *ſober* voice you hear,
Contracts with *bigot* frown her ſullen brow;
Caſts round RELIGION's orb the miſts of *fear*,
Or ſhades with horror, what with *ſmiles* ſhould glow:

No.—SHE would warm you with *ſeraphic fire*;—
Heirs as you are of HEAVEN's eternal day;—
Would bid you *boldly* to THAT HEAVEN *aſpire*,
Not ſink and ſlumber in yon cells of clay!—

Is this the *bigot*'s rant?—Away, *ye vain!*
Your doubts, your fears, in *gloomy dulneſs* ſteep:
Go!—*ſoothe* your ſouls in ſickneſs, death, or pain,
With the ſad ſolace of eternal ſleep!

Yet know, *vain ſceptic!* know, th' ALMIGHTY MIND,
Who breath'd on *Man* a portion of HIS fire;
Bade his *free ſoul*—by earth nor time confin'd—
To HEAVEN; to IMMORTALITY; aſpire!

Nor ſhall this *pile of hope*, HIS BOUNTY rear'd,
By *vain Philoſophy* be e'er deſtroyed;
ETERNITY, by ALL or *hoped* or *fear'd*,
Shall be by ALL or *ſuffer'd* or *enjoy'd*.”

common with them), and wherein their *diſparity* conſiſts. Thus, our knowledge is vaſtly greater than *the ſum* of what all it's objects ſeparately could afford; and when a *new* object comes within our reach, the addition to our knowledge is the greater, the more we already know: ſo that it increaſes, not *as* the new objects increaſe, but in a *much higher proportion*." * * *

156. And was the exalted *mind*, that reaſoned thus powerfully, and triumphed over *bodily* infirmity in the very jaws of *death*, *material?*—Was all this merely the reſult of corporeal *mechaniſm?* Was the profound proceſs of this noble argument for the ſurvival and improvement of the ſoul; the grand *climax* at the beginning, riſing from this globe to the milky way; ſuch clear-ſighted intelligence of the maſter principles and lofty affections, and aſpiring views of *human nature*, in the ſequel; ſuch ſubtile and abſtruſe analyſis of the doctrine of *combinations* and of the *fluxionary* calculus, applied at it's unfiniſhed cloſe; ſuch humble, yet aſtoniſhing penetration and inſight into the myſterious ways of Providence, in this life—"*Looking thro'* NATURE *up to Nature's* GOD:"—while the quivering lamp of light, thus darting the brighteſt flaſhes of celeſtial fire, was expiring in the ſocket.—Was all this admirable aſſemblage, of the pious and the profound, the ſublime and the beautiful, the argumentative and the eloquent, no other, ye *Hartleys!* ye *Prieſtleys!* ye *Condorcets!* ye *Godwins!* or ye *Darwins!* than mere animal mechaniſm? wrought and effected, wholly and ſolely, by *vibratiunculæ* or minute tremors of the *pulpy* ſubſtance of the brain!!! (*f*)

> "O curvæ in terras animæ et cœleſtium inanes!
> Quid juvat hoc, *templis* noſtros immittere *mores*;
> Et *Bona* DIS ex hâc *ſceleratâ* ducere *pulpâ?*" PERSIUS.

D'Alem-

(*f*) The ſublime Philoſophy of HOLY WRIT, and that alone, has *authoritatively* decided the myſterious and momentous queſtions of the *immateriality* of the *Soul*, of it's natural *mortality*, and of it's *reunion* with an *incorruptible body* after death; recording the *Origin* and the *End* of the *Human Race*:—

1. "THE LORD OF GODS formed *the Man*, *duſt* of the ground,
And breathed into his noſtrils, *breath of life*;
So *Man* became a *living ſoul*." *Gen.* ii. 7.

2. "*Duſt* thou art, and unto *duſt* ſhalt thou return." *Gen.* iii. 19.

3. "All *Fleſh* ſhall periſh together;
And *Man* ſhall turn again unto *duſt*." *Job* xxxiv. 15.

4. "I know that my DELIVERER is LIVING;
And at the *laſt* [*day*] will *ariſe* [in judgment] upon *duſt*:
Yet, after this *ſkin* be mangled,
Even in my *fleſh* ſhall I perceive GOD;
Whom I ſhall perceive, *on my ſide*,
And mine eyes ſhall ſee [HIM] *not eſtranged*." *Job* xix. 25.

5. "The

D'Alembert's Mournful Reflections.

157. As a terrific contraſt to the death of the *Chriſtian Philoſopher*, I ſhall propoſe the dreadful gloom that overhung the latter days of a *philoſophizing infidel*, furniſhed by his correſpondence with the King of *Pruſſia*, another of the original conſpirators againſt Chriſtianity in the train of *Voltaire*.—*D'Alembert* —at once the glory and diſgrace of the French *Academy of Sciences*—"*deceiving*" others, by his perfidious ſophiſtry, but not fully "*deceived*" himſelf—ſo as to conquer and blunt the ſtings of remorſe, and the terrors of a guilty conſcience, anticipating "the juſt reward" of thoſe fiends in human form, who,

5. "The *duſt* ſhall return to *the earth* as it *was*;
But *the ſpirit* ſhall return to God who gave it." *Eccleſ.* xii. 7.

6. "Jesus Christ—" illuſtrated *life* and *incorruption*." 2 *Tim.* i. 10.
"Himſelf—" *the firſt fruits* of the *ſleepers*." 1 *Cor.* xv. 20.

7. "Lo! I tell you a *myſtery*:
We ſhall not all *ſleep*, but we ſhall all be *changed*;
In a moment, in the twinkling of an eye,
At the *laſt trumpet*."——

8. "For this *corruptible* (body) muſt put on *incorruption*;
And this *mortal* (ſpirit) muſt put on *immortality*."

9. "O *Death!* where is thy *ſting* [in the *body*]?
O *Hades!* where is thy *victory* [over the *ſoul*]?

10. "The *ſting* of *Death* is *Sin*———
But thanks be to God, who giveth us the *victory*,
Through our Lord Jesus Christ." 1 *Cor.* xv. 51—57.

11. "I am the *Firſt* and the *Laſt* and the *Living*,
And I became *dead*; and lo! I am *living*
For ever and ever. Amen.
And I hold the keys of *Hades* and of *Death*." *Rev.* i. 17.

12. "I ſay unto you, *my friends*:
Be not afraid of them who kill the *body*,
But are not able to kill the *ſoul*;
But be afraid rather of Him, who is able
To deſtroy both *ſoul* and *body* in *hell*." *Matt.* x. 28.

13. "Who will transform the *body of an humiliation*,
To become the reſemblance of the *body of his glory*;
According to the energy of his ability,
Even *to ſubject all things unto himſelf*." *Phil.* iii. 21.

who, after zealouſly labouring through life to undermine the *hopes* of others, too fatally ſucceed at length in blaſting their own!

In one of his laſt letters, publiſhed by the ingenious, the unprincipled, the execrable, and the wretched, *Condorcet*; who beſet *D'Alembert* when expiring, and boaſted, that, if he had not, "*D'Alembert would have flinched alſo*,"—like their grand maſter *Voltaire*—in all his dying horrors and agonies, "*worſe than the furies of Oreſtes*"—the peeviſh and gloomy philoſophiſt thus complains to his royal correſpondent, *Frederick*:

—"Study *ſometimes* engages me, and converſation *ſometimes* entertains me; but I am ſoon fatigued with either; and am no ſooner left to myſelf than my *uneaſy reflections* recur, and my *ſolitude* again frightens and freezes me. In this condition, I reſemble a man who ſees before him *a long and dreary deſart*, which he *muſt* paſs; and at the end of that *melancholy proſpect*, the *abyſs of deſtruction* open to receive him: without finding, at the brink of that *hideous chaſm*, a ſingle perſon that will be *afflicted* with his downfal, or that will even *remember* his exiſtence, when he has ſunk into *endleſs perdition!!!*"

From SUCH *Philoſophy*, GOOD LORD! *deliver us.*

158. How utterly falſe, then, and unfounded are the viſionary ſpeculations of the votaries of "*a miſtreſs that never grows grey!*" (if we liſten to the illi-

14. "For *fleſh* and *blood* [i. e. an *earthly* body] is not able
To inherit GOD's kingdom; neither ſhall *corruption*
Inherit *incorruption*." 1 *Cor*. xv. 50.

15. "The maſs of *the ſleepers*, [though] *duſt of the earth*,
Shall awake; ſome to everlaſting *life*,
But ſome to ſhame and everlaſting *contempt*:
When the *ſages* ſhall ſhine, as the *brightneſs*
Of the firmament; and the *juſtified* of the many,
As the *ſtars* for evermore." *Dan*. xii. 2.

16. "As *one* ſtar differeth from *another* ſtar
In *glory*." 1 *Cor*. xv. 41.

17. "In my FATHER's houſe are *many manſions*." *John* xiv. 2.

Such are the awful, the animating, the ennobling doctrines of *Patriarchal* and *Evangelical* WISDOM—*revealed*, "at ſundry times and in divers manners," to his *rational* creatures, by the FATHER OF LIGHTS, through 'THE SON OF HIS LOVE'—THE ORACLE or Expounder of his Decrees, under the *Patriarchal* and *Chriſtian Economy*. Nor can this important ſelection of ſtrictly harmonizing *Texts* be deemed irrelevant or unſeaſonable by the *Mathematician* or the *Philoſopher*, who would wiſh zealouſly to ſtem that Torrent of *Infidelity*, which is the diſgrace of a boaſted AGE OF REASON:

Diſcite, O miſeri! et *cauſas* cognoſcite rerum:
Quid *ſumus*, et quidnam *victuri* gignimur; *Ordo*
Quis datus;———quem *Te* DEUS eſſe
Juſſit.——— PERSIUS.

terate *Paine*'s eulogy on SCIENCE.)—And how wretched, how deluſive the *happineſs* ſhe can afford, detached from RELIGION—that true miſtreſs of the *wiſe* and *good*—whoſe humble handmaid SCIENCE ought to be.

—"*To be happy in old age*," ſays *Paine*, "it is neceſſary that we accuſtom ourſelves to objects that *can* accompany the mind *all the way through life*; and that we take the reſt as good in their day. The mere man of *pleaſure* is miſerable in old age; and the mere drudge in *buſineſs* is but little better: whereas *Natural Philoſophy*, *Mathematical* and *Mechanical Sciences*, are a continual ſource of *tranquil pleaſure*. And, in ſpite of the gloomy dogmas of *Prieſts* and of *Superſtition*, the ſtudy of theſe things is the ſtudy of the *True Theology*; it teaches man to *know* and *admire* the CREATOR: for the *principles of Science* are in the *Creation*, are *unchangeable* and of *divine origin*."

159. Alas! what a wretched happineſs did *Science* procure *D'Alembert*, who, at the cloſe of his days, thus found to his coſt, that he had been all his life long embracing a *cloud* for a *goddeſs!*—And that "the ſtudy of *theſe things* is not indeed the ſtudy of the *True Theology*," we learn from the monſtrous *errors* of ancient and modern *Sophiſts*; which led them to *forget* and to *vilify* the CREATOR: "The *principles of Science* are *not* indeed in the Creation;"—they lie much deeper than the *viſible* world; and are only to be learned from REVEALED WISDOM and OMNISCIENCE:—whoſe *Sovereign Will* is the *Law* of the Univerſe; *changeable* at pleaſure, though in HIMSELF "*unchangeable*." *James*, i. 17.

160. Theſe principles, the lowly and unaſſuming ſpirit of *Primæval* and *Patriarchal Science* endeavoured cautiouſly to trace and develope, amidſt "the *obſcurity* that prevails in the midſt of things;"—humbly and devoutly thanking the FATHER OF LIGHTS for thoſe faint *glimpſes* of ETERNAL TRUTHS, which, from time to time, and from ſeaſon to ſeaſon, He vouchſafes to reveal to thoſe truly Philoſophic Students, who, in "an *honeſt and good heart*," *hear*, ſeek, embrace, and keep the *Word of God*, whether communicated by *Revelation*, or thence unfolded by *Reaſon*: and earneſtly and diligently, and devoutly, ſtrive to *bring forth fruit with patience*.

"*Cœlum* quippè ac *Terra*, ac *Mare*, omníſque *Creatura* quæ videri atque intelligi poteſt, ad hanc præcipuè diſpoſita eſt *humani generis* utilitatem, ut *Natura Rationalis* de contemplatione tot ſpecierum, de experimentis tot bonorum, de perceptione tot munerum, ad cultum et dilectionem SUI imbueretur AUCTORIS; *implente omnia* SPIRITU DEI, "*in quo vivimus, movemur, et ſumus*."—"Adhibita ſemper eſt univerſis hominibus *quædam* ſupernæ menſura doctrinæ, quæ [licet] *parcioris occulticríſque gratiæ*, ſufficit tamèn (ſicùt DOMINUS judicavit), quibuſdam ad remedium, omnibus ad *teſtimonium*.—Ait *Plotinus*: "*Si* MUNDI *vocem audiremus*, nihil aliùd eum dicere quam DEUS ME FECIT; *non Cretenſis Jupiter aut Arcas Mercurius, ſed* DEUS *ille* ΑΓΝΩΣΤΟΣ [*neſcibilis*], de quo *Paulus* ad *Athenienſes*." *De Vocatione Gentilium*.

Vide poſthàc, § 205, Not. (*p*).

Scriptori huic egregio, veriſſimè de Dei cognitione, quatenùs ex rationibus *phyſicis* eam colligere fas ſit, ſuffragatur ſana *Newtoni* ſententia: *Princip.* p. 529.

"*De* DEO utique ex *phænomenis* diſſerere ad PHILOSOPHIAM NATURALEM *pertinet.*"

161. Placet igitùr, quoniam omnis diſquiſitio de *cauſis* et *effectis* naturalibus ad agnitionem CAUSÆ PRIMARIÆ perducere debet, "quæ *minimè materialis* eſt," *Newtono* judice—VINDICIAS quaſdam THEOLOGICAS *Sapientum Priſcorum*, atque *Newtoni* horum cultoris, ſubjungere.

De Philoſophiâ Atomicâ Priſcorum Chaldæorum atque Hebræorum.

162. Nec minùs injurioſè invehitur *Robiſon* in *Philoſophiam Atomicam*, qualem excoluit *Newtonus*; quæ minimè confundenda eſt cum *Epicureâ* iſtâ nupèr redivivâ, Rationibus Metaphyſicis atque Mechanicis rerum naturalium cauſas tribuente, et CAUSÆ PRIMARIÆ oblitâ. Hanc enim jure perſtringit ipſe *Newtonus* in *Optics*, p. 343, his verbis:

"And for rejecting ſuch a *medium* (as is totally denſe or full of matter), we have the authority of the oldeſt and moſt celebrated Philoſophers of *Greece* and *Phœnicia*, who made a *vacuum* and *atoms* and the *gravity* of atoms the firſt principles of their philoſophy: tacitly attributing gravity to *ſome other cauſe* than denſe matter."

"Later Philoſophers baniſh the conſideration of *ſuch a cauſe* out of *Natural Philoſophy*; feigning *hypotheſes* for explaining all things *mechanically*, and referring other cauſes to Metaphyſics: whereas the main buſineſs of Natural Philoſophy is to argue from *phænomena*, without feigning hypotheſes, and to deduce *cauſes* from effects, till we come to the VERY FIRST CAUSE, which certainly is not *material*."

Hæc brevitèr effata et delibata tantùm, fuſiùs exponere, atque ex philoſophiâ primâ confirmare, jam libet:

163. Priſcorum Chaldæorum atque Hebræorum, Philoſophia phyſica erat *Atomica*, Coſmogoniæ *Moſaicæ* conſentanea. Hanc invexerunt in Thraciam, *Orpheus*, ille "*Sacer interpreſque Deorum*," qui undecim generationes antè Trojæ excidium floruit, teſte *Suidâ*, ſeu annis $366 + 1184 = 1550$ circitèr antè Æram Chriſti Vulgarem; in Ioniam et Græciam, *Thales*, "*phyſicorum princeps*," *Pherecydis* Syri diſcipulus; in Italiam, *Pythagoras*.—Lucida ſidera omnes.

Quanto fuerunt in honore Sapientes *Chaldæi* et *Hebræi*, apud *Græcos* vetuſtiores, diſcimus ex reſponſo *Oraculi Pythici*, quibuſdam ſciſcitantibus, *Ex gentibus quinam omnium ſapientiſſimi?*

Μουνοι Καλδαιοι σοφιην λαχον, ηδ᾽ αρ᾽ ῾Εβραιοι,
Αυτογενητον Ανακτα σεβαζομενοι ΘΕΟΝ ΑΥΤΟΝ.

"Soli *Chaldæi* ſapientiam ſortiti ſunt atque *Hebræi*,
Per-ſe-genitum Regem venerantes DEUM IPSUM."

164. Præclariſſimum extat teſtimonium in fragmentis *Verſuum Orphicorum*, ab *Onomacrito* congeſtis, de Decalogo Sacro, apud Montem *Sinai* promulgato, annis 1649 circitèr ante *Ær. Chriſt.* (ſecundùm emendationem noſtram *Sacræ Chronologiæ*, poſthàc forsàn edendam).

Αρχην ΑΥΤΟΣ εχων, ἁμα και Μεσον, ηδε Τελευτην·
῾Ως λογος Αρχαιων· ὡς ῾Υδογενης διεταξεν,
Εκ ΘΕΟΘΕΝ γνωμαισι λαβων κατα διπλακα Θεσμον.

"*Principium* IPSE habens, ſimùl et *Medium*, atque *Finem*:
Ut ſermo Priſcorum; ut *Aqua genitus* præcepit,
E DEO ſententiis quum accepiſſet *duplicem Legem*."

Præclara hæc deſcriptio DEI IPSIUS optimè conſentit cum *Scripturis Sacris*. *Iſa.* xli. 4. *Rev.* i. 8. &c. &c.

"HIM *Firſt*, HIM *Laſt*, HIM *Midſt*, and *without End*." *Milton.*

Vox ῾Υδος, antiquitùs pro ῾Υδωρ, "*aqua.*"—*Heſychius.* Per ῾Υδογενης igitùr elegantèr expreſſit *Orpheus* vocem Hebræam *Moſeh* (vel *Moſes*), quæ ſignificat "ex aquâ *ereptum.*"—*Exod.* ii. 10. Et per διπλακα Θεσμον, "duplicem *Decalogi* Tabellam"—"digito DEI deſcriptam." *Exod.* xxxii. 26. et xxxiv. 1. (*g*). Atque hoc *Orphei* teſtimonium expreſſiſſe videtur *Juvenalis*, *Sat.* xiv. 102.

"Tradidit arcano quodcunque volumine *Moſes*."

165. Alterum

(*g*) Hujúſce eximii *Fragmenti Orphici*, ab *Onomacrito* conſervati, auctoritatem inſectatur doctiſſimus *Cudworth*, ὁ πλατωνιζων, ſed hypercriticè potiùs, ut videtur, *Intellectual Syſtem*, p. 300.

"There ſeem to be ſome *Orphic* Verſes ſuppoſititious, as well as there were *Sibylline*, &c.—Cæterùm cum híſce probè conſentiunt verſus *Orphici* quos ipſe *Cudworth* pro genuinis agnoſcit, et ex *Proclo* citat, p. 301.

ΖΕΥΣ πρωτος γενετο, ΖΕΥΣ ὑςατος αργικεραυνος,
ΖΕΥΣ κεφαλη, ΖΕΥΣ μεσσα, ΔΙΟΣ δ᾽ εκ παντα τετευκται.

"JOVIS *primus* exiſtebat, JOVIS *ultimus* altinonans,
JOVIS *caput*, JOVIS *medius*, à JOVE *omnia fabricantur*."

Ex

165. Alterum, quoque, nec minùs inſigne teſtimonium de *Chaldæorum* divinatione profert *Orpheus.*

> Ου γαρ κεν τις ιδοι θνητων μεροπων ΚΡΑΙΝΟΝΤΑ,
> Ει μη μουνογενης τις, απορρωξ φυλου αωθεν
> Χαλδαιων· Ιδρις γαρ εην Αστροιο ϖορειας.

> " Nullatenùs enìm quiſquam mortalium articulatè loquentium viderit Regem,
> Niſi *dilectus* quidam antiquo ab origine gentis
> *Chaldæorum* ; peritus enìm erat *aſtri exortûs.*"

Referunt hoc vulgò interpretes ad *Abrahamum,* " quem omnis vetuſtas Orientis ſummum aſtrologum fuiſſe crediderit." *Scaliger* de *Emend. Temp.* Not. in *Fragment.* p. 48 ; et *Cudworth,* p. 300. Sed manifeſtò reſpicit vatem Chaldæum, *Balaam, Moſis* co-ævum, " qui apud *Peth-Ur* (ſeu Hebräicè *Beth-Ur,* " Domus Lucis,") juxtà fluvium *(Tigrim)* degebat," (*Numb.* xxii. 5.) ; et qui inſigne illud vaticinium cecinit de *Shilo, Jacobi* progenie, (*Gen.* xlix. 10.) ſeu de *Meſſiâ* " Rege," *Num.* xxiv. 17.

> " *Video* Illum, *ſed non nunc* ;
> *Cerno* Illum, *ſed non propè* ;
> *Prodibit* Astrum *ex Jacobo,*
> *Exorietur* Sceptrum *ex Iſräele.*"

Cujúſque poſteri, Μαγοι απο ανατολων, " *Magi Orientales,*"—actu " *viderunt aſtrum ejus in exortu ſuo,*" (εν τη ανατολη,) 1600 annis poſteà, è longinquo conſpicati, *nocte* forsàn *natali* Chriſti. Confer *Matt.* ii. 1—10, et *Luc.* ii. 9—15.

166. Priſcos *Chaldæos* atque *Hebræos vacuum* et *atomas* admiſiſſe in coſmogoniam ſuam, colligere licet ex *Virgilio,* mythologiæ antiquæ obſervantiſſimo,

---

Et verſum ſecundum, levi variatione, ex ipſo *Plutarcho* quoque citat, p. 305.

> ΖΕΥΣ αρχη, ΖΕΥΣ μεσσα, ΔΙΟΣ δ' εκ παντα τελονται.
> " Jovis *principium,* Jovis *medius,* à Jove *omnia fiunt.*"

Ubi pro κεφαλη citàntis *Procli, Plutarchus* vocem ſynonymam αρχη citat ; utramque proculdubio à voce *Hebræâ,* ראש, vel ראשית ambiguè " *caput* " vel " *principium* " deſignante. *Gen.* i. 1.—Vide quoque Not. (*n*).

Necnon ex *Timotheo* chronographo quaſi *Orphicæ Theologiæ* foret ; p. 305.

> Δια της ΘΕΟΤΗΤΟΣ ϖαντα εγενετο,
> Και ΑΥΤΟΣ εστι ϖαντα.

> " Per Deitatem omnia facta ſunt,
> Et Ipse eſt Omnia."

Veruntamèn fatendum eſt quòd *Orpheus* ipſe falſa veris immiſcuit. Refert enim *Juſtinus Martyr* ; Πολυθεοτητος ϖατηρ και ϖρωτος διδασκαλος—quòd " *Polytheiſmi pater atque præceptor primus fuit*" *Orpheus.* Vide *Cudworth,* Int. Syſt. p. 298.

ubi *vinoſum Silenum*—"*Cuſtodem famulúmque Dei (Bacchi) alumni*," et proinde ex Patriarchis forsàn vetuſtiſſimum, (inſtar *Noah*, vino oppreſſi, *Gen.* ix. 21—24.) ità vaticinantem inducit, *Eclog.* vi. 31.

"Námque canebat, utì MAGNUM per INANE coacta
Semina *terrarúmque*, *animaéque*, *maríſque* fuiſſent,
Et *liquidi* ſimùl *ignis:* ut his exordia primis
Omnia; et ipſe tener Mundi *concreverit* orbis."

167. Paritérque *Euripides*, *Anaxagoræ* diſcipulus, traditionem vetuſtiſſimam refert, in fragmento *Menalippes*, *Barnes Eurip.* p. 481, citato à *Diodoro Siculo*, I.

Ουκ εμος ὁ μυθος, αλλ᾽ εμης μητρος ϖαρα·
Ὡς ουρανος τε γαια τ᾽ ην μορφη μια·
Επει δ᾽ εχωρισθησαν αλληλων διχα,
Τικτȣσι ϖαντα, κ᾽ ανεδωκαν εις φαος,
Δενδρα, ϖτηνα, ϑηρας, ȣ̔ς ϑ᾽ ἁλμη ϑρεφει,
Γενος τε ϑνητων.

"Nec meus hic ſermo, ſed quem præcepit Mater:—
Ut cælum et terra erant forma una;
Ubi verò à ſe invicèm ſejuncta ſunt,
Pariunt omnia, ediderúntque in lucem,
Arbores, *aves*, *feras*, [*piſcéſque*] quos alit mare,
Genúſque mortalium."

Manifeſtò deſumuntur hæc *mythica* ex Coſmogoniâ *Moſaicâ*; de ſtatu terræ *chaotico*, et generatione *vegetabilium*, *animalium* atque *hominum*, ordine *Moſaico* diſſerentia; cæterùm in hoc peccant quod SUMMI OPIFICIS vim creatricem vix, et ne vix quidem, reſpiciunt, ſed productionem rerum *ſpontaneam* innuunt.

168. Atqui híſce excerptis, ſeminarium habes *Philoſophiæ Atomicæ* qualem excolebant Sapientes Scholæ *Chaldææ primævæ* ac *Hebrææ*, antequàm eam peſſundederunt atque inquinaverunt pravè philoſophantes, *Magi*, *Brachmanni*, &c. Orientales; atque *Stoici*, *Epicuræi*, &c. Occidentales, ingenti veræ Theologiæ atque Philoſophiæ detrimento.

169. Et egregiè diſtinguit *Plutarchus*, inter "*valdè antiquos theologos atque poëtas*"—quales fuerunt *Orpheus* ejúſque diſcipuli, qui "JOVEM *principium*, JOVEM *medium*, à JOVE *omnia fiunt*"—præceperunt, et "*híſce juniores, qui Phyſici ſunt appellati*"—quales poſt *Thaletem* exſtiterunt *Anaximander*, *Anaximenes*, &c. Scholæ *Ionicæ*; atque *Itali* iſti degeneres Scholæ *Pythagoreæ*, *Leucippus*, *Democritus* et *Epicurus*, qui, "*ab hoc principio honeſto atque divino temerè palantes, cauſam*

omnium primariam in corporibus, corporeiſque affectionibus, pulſibus, mutationibus commixtionibúſque poſuerunt." —Undè Φυσικοι et Αθεοι (" *Naturaliſts*" and "*Atheiſts*") indifferentèr, et quaſi per nomina ejuſdem ferè ſignificationis, vulgò ſunt appellati. *Cudw.* p. 305.

170. Etiàm *Anaxagoras* ipſe, appellatus Νȣς, "*Mens,*" κατ’-εξοχην, qui priſcam notionem MENTIS SUPREMÆ revocavit, "Ejus opificis in mundo condendo, rationibúſque phyſicis, nullatenùs eſt uſus, ſed ad cauſas rerum merè mechanicas, *aëres, ætheres*, et *aquas* minimè idoneas, aliáque prorsùs abſurda, confugit," ità querente *Socrate*, in *Phædone*.

171. Hinc anſam vituperandi arripiens *Ariſtoteles*, totam *Phyſicam Atomicam* inſectabatur præceps, ex rationibus ſuis *Metaphyſicis*, multa ſomnians de *quinto genere elementorum è quo fiant mentes et aſtra*, de *fugâ vacui*, de *pleno univerſali*, de *æternitate mundi*, de *formis ſubſtantialibus* (ſeu εντελεχειαις). Inanitèr autèm atque arrogantèr gloriatur—"*Se videre, quod paucis annis magna acceſſio facta eſſet; brevi tempore, Philoſophiam planè abſolutam fore.*"—*Cicero, Tuſcul.* I. 10; III. 28. *Acad.* I. 4.

172. *Ariſtotelis* faſtum jactantiámque ſecuti, *Carteſiani* et *Leibnitziani*, ingenti ſcientiarum detrimento, Phyſicorum Græcorum *Mechanicam* irrationalem et phantaſticam recolebant; *vortices, monadas*, &c. temerè fingentes: donec modeſtum *Newtoni* ingenium priſcas revocabat artes; atque à Logomachiis averſus, ad Diſciplinam Scholæ *Ionicæ, Antiquæ Hebrææ* atque *Chaldææ*, ſerò tandèm redibat, Phyſicam *Atomicam* inſtaurabat, eam fundamentis ſolidis atque inconcuſſis *Inductionis* atque *Experimentorum* ſtabiliendo.

—"*Hypotheſes* non fingo: Quicquid enìm ex *phænomenis* non deducitur, Hypotheſis vocanda eſt. Et Hypotheſes, ſeu *Metaphyſicæ*, ſeu *Phyſicæ*, ſeu *Qualitatum occultarum*, ſeu *Mechanicæ*, in PHILOSOPHIA EXPERIMENTALI locum non habent."
Principia, pag. ult.

173. Quàm injurioſè igitùr invehitur Profeſſor *Robiſon* in *Philoſophiam Atomicam*, qualem excolebant priſci *Chaldæi* et *Hebræi*, eorúmque Diſcipuli; et qualem inſtaurabat *Newtonus*, ſobriam, ſanam, *principiis Mathematicis* reconditioribus et ſubtilioribus forsàn quam pro captu tyronum, imò Mathematicè doctorum, (etiàm *Profeſſorum* forsàn), ſtabilitam atque demonſtratam. *Logicam* certè parùm ſapit, *ex abuſu* alicujus rei, artis vel ſcientiæ, *uſum* detrectare; et *Mechaniſmum* adeò irrationalem, abſurdum et impium Recentiorum Philoſophiſtarum, ex *Mechanicâ Rationali Newtonianâ* deducere.

De Æthere Antiquo, Deo Ficto Gentilium.

174. Vocem ipſam *Æther* mutuatus eſt *Newtonus* ex Philoſophiâ Priſcâ, ità referente *Cicerone.* N. D. II. 33.

"Et cùm quatuor ſint genera corporum, *viciſſitudine* eorum mundi continuata natura eſt. Nam ex *terrâ*, aqua; ex *aquâ* oritur aër; ex *aëre*, *æther*: Dein retrorsùm viciſsìm, ex *æthere*, aër; ex *aëre*, aqua; ex *aquâ*, *terra infima*, &c."

Exemplum hìc habes *Compoſitionis* et *Reſolutionis* antiquiſſimæ, ſecundùm quas "*mundi partium conjunctio continetur*"—ex mente Phyſicorum priſcorum, ſanum, Cantui *Sileni* atque *Coſmogoniæ Moſaicæ* accommodatiſſimum: ubì *æther* manifeſtò reſpondet "*liquido igni*," quarto nimirùm elemento, (hodiè, *fluido electrico*) ſubtiliſſimo, omnium corporum poros et interſtitia liberrimè permeanti, quod inter potentiſſimos naturæ agentes jure recenſetur; et cujus identitatem cum *igne cœleſti (lightning)* detexit et confirmavit, noſtro tandèm ævo, curioſa ſedulitas ſagaciſſimi Naturæ indagatoris, *Benjamini Franklin*.

Vox αιθηρ quoque, ſi compoſitionem ejus reſpicias, derivatur ex αιθω, "*uro*," et αηρ, "*aër*," ſeu *atmoſphæra* (quæ forsàn ex אור *aur*, "*lux*," ultimò deducenda eſt). Et quidèm ab *Heſychio* definitur Αιθηρ, ὁ ὑπερ τα νεφη τοπος, "*regio ſuprà nubes*;"—atque ſignificatione alterâ ſynonymâ, εμπυρισμον, "*inflammationem*."

175. Pro ſymbolo (*h*) certè "Dei *lucem inacceſſibilem habitantis*" primitùs, Patriarcharum ſæculis, *Job*, *Abraham* atque "*Chaldæorum* progenitorum, *Aur*, (i. e.

(*h*) Exordium Elegiæ à reverendo viro et poetâ docto et diſerto, *Thomâ Maurice* ſcriptæ in obitum celeberrimi et optimi viri, *Gulielmi Jones*, Indorum Apoſtoli, Chriſtianique Philoſophi, pulchrè et ornatè deſcribit patrias ſedes *Chaldæorum*, et primævam diſciplinam *Magorum*.

"Where the dark cliffs of rugged *Taurus* riſe,
From age to age by blaſting lightnings torn,
In glory burſting from the illumin'd ſkies,
Fair *Science* pour'd her firſt auſpicious morn.

The hoary *Parthian* Seers, who watch'd by night
Th' eternal Fire in *Mithra*'s myſtic cave,
(*Emblem* ſublime of that PRIMÆVAL LIGHT
Which to yon ſtarry orbs their luſtre gave,)

Exulting, ſaw it's gradual ſplendours break,
And ſwept, ſymphonious, all their warbling lyres,
'Mid *Scythia*'s frozen glooms THE MUSES wake,
While happier *India* glows with all their fires."

(i. e. "*Lux*,") incolentium," *Gen.* xi. 28. uſurpatus eſt Αιθηρ, ſeu "*Aër* ille *igneus*."

176. *Theologiæ Chaldææ* præclariſſimum extat ſpecimen, apùd *Suidam*, de *Orpheo* ità diſſerentem :

Εφησε δε, ὁτι ΦΩΣ, ῥηξαν τον Αιθερα, εφωτισε την Γην, και πασαν Κτισιν· Εκεινο ειπων "ΤΟ ΦΩΣ" το ῾ΥΠΕΡΤΑΤΟΝ παντων, το "ΑΠΡΟΣΙΤΟΝ," το παντα ΠΕΡΙΕΧΟΝ· ῾Οπερ ωνομασε ΒΟΥΛΗΝ, ΦΩΣ, ΖΩΗΝ.

Ταυτα τα τρια Ονοματα ΜΙΑΝ ΔΥΝΑΜΙΝ απεφηνατο και ῾ΕΝ ΚΡΑΤΟΣ του ΔΗΜΙΟΥΡΓΟΥ παντων ΘΕΟΥ, του παντα εκ του μη οντος παραγαγοντος εις το ειναι, ὁρατα τε και αορατα.

"Fatus eſt *Orpheus*, LUCEM, rupto *Æthere*, Tellurem illuſtrâſſe, omnémque Creationem : Illam dicens LUCEM, *ſupremam* omnium, "*inacceſſibilem*," omnia *continentem* ; quam nominavit CONSILIUM, LUCEM, VITAM."

"Hæc tria nomina declarant UNAM POTENTIAM, UNUMQUE DOMINIUM OPIFICIS *omnium* DEI, qui adduxit omnia *ex non exiſtente in exiſtentiam, viſibilia paritèr et inviſibilia*."

177. Mirus ſanè et ferè incredibilis conſenſus hic obſervandus cum Libris Evangelicis —Nam "DEUS eſt LUX," 1 *John*, i. 5. "*Lucem habitans inacceſſibilem*," 1 *Tim.* vi. 16. Et JESUS CHRISTUS eſt "LUX *Mundi*," *John* viii. 12. Ut et "SAPIENTIA DEI," *Luk.* vii. 35. et *Matt.* xi. 19.—itémque "VITA," *John* i. 4. et xiv. 16. Et declaravit CHRISTUS, Εγω και Πατηρ ῾ΕΝ εσμεν.—*Orpheo* nimirùm optimè interpretante—῾ΕΝ ΚΡΑΤΟΣ—"PATER et EGO *unum ſumus* [DOMINIUM]." *John* x. 30.

Et *Milton*, adhùc ſublimiùs et eruditiùs, varias ſententias veterum Philoſophorum exponit, in celeberrimâ illâ LUCIS invocatione, P. L. III. ſub initio :

"Hail, HOLY LIGHT! Offspring of Heaven, Firſt-born,
Or of th' eternal co-eternal Beam ;
May I expreſs thee unblam'd! Since "GOD IS LIGHT,"
And never but in "*unapproached light*
Dwelt" from Eternity, dwelt then in Thee,
Bright *Effluence* of bright ESSENCE INCREATE :
Or hear'ſt thou, rather, pure *Ætherial Stream*,
Whoſe fountain who ſhall tell? Before *the Sun*,
Before *the Heavens* thou wert, and at the voice
Of GOD, as with a mantle, didſt inveſt
The riſing *World of Waters*, dark and deep,
Won from the *void* and *formleſs* Infinite."

Notandum quòd per *Mithram* intelligebant *Parthi Magi* antiquiſſimi non *Solem* (ut malè poſteri), ſed Sole ſuperiorem ; quem appellarunt, Ὁ κρυφιος Θεος, "*Deus abſconditus*,"—Παντων Ποιητης και Πατηρ, "*Omnium Opifex et Pater*." *Cudworth*, p. 286. Quibuſcum optimè conſentit *Job* xxiii. 9. *Judg.* xiii. 18. *Iſa.* ix. 5. *Iſa.* xlv. 15. Vide § 179.

178. Sana et ſublimis *Noachidarum* Theologia peſſundari et corrumpi incœpit circà quintam à Diluvio generationem; et potiſſimùm à *Nimrod*, *Cuſhitarum* principe, qui cultum aſtrorum primus introduxit; etiàm proverbio celebratus, ut "*Ingens ille venator, coram Domino*;" qui poſt mortem ejus, à ſubditis ſuperſtitioſis, tranſlatus eſt in aſtrum *Orionis* cum canibus (qui *Sirius* et *Canicula* ſunt appellati), quaſi in cœlis etiamnùm venaturus *Urſam Majorem* ejúſque Catulos, Circulo *Arctico* commorantem, et motu diurno revolventem; quam vivus inſectabatur in Montibus *Aſſyriæ*, *Ararat* (*i*): Nam hic bellator acerrimus quoque, cum ab Oriente migraturus ſit in ſedem ab oraculo ſuis deſtinatam, ſubſtitit rebellis " in amœnâ terrâ *Shinaar*, ubi principium dominationis ſuæ poſuit *Babel*, ſeu *Babylonem*; et indè egreſſus, invaſit *Aſſyriam* (ſedem *Noachidarum* poſt Diluvium, et *Shemitæ Peleg*, in diſtributione divinâ terrarum, allocatam), et *Niniven* condidit." Confer *Gen*. x. 8—12. et xi. 1—8. et *Joſephum*.—Colebatur hic *Rebellis* ſub nomine *Bel* (à *Heb*. בעל *Baal*, "Magiſter,") ab Aſſyriis, undè Græcorum *Belus*; cujus nomen proprium forsàn erat *Nin*—(i. e. נין "*Filius*,") (*k*)—utpotè clariſſimus filiorum *Chus*; et quidem appellatur ipſa *Ninive* ab *Herodoto*, paſsìm, ἡ Νινος, "Civitas *Nini*."—Fuit autèm Pronepos *Chus*, per intermedios *Raamah* et

---

(*i*) Hujuſmodi venationem pulchrè deſcribit *Homerus*, *Il*. xviii. 485.

Αρκτον δ' ἡν και Αμαξαν επικλησιν καλεουσιν·
Ἡ τ' αυτο στρεφεται και τ' Ωριωνα δοκευει.

"*Urſámque*, quam cognomine *Plauſtri* appellant [*Ægyptii*],
Quæ ibidèm ſeſe vertit, obſervátque *Orionem*"

—quaſi illum et canes venaticos metuens.

Atque iterùm *Orionem*, in Infernis, ſimilitèr occupatum introducit, *Odyſſ*. xi. 571.

Τον δε μετ' Ωριωνα πελωριον εισενοησα
Θηρας ὁμου ἐιλευντα κατ' ασφοδελον λειμωνα,
Τους αυτος κατεπεφνεν εν οιοπολοισιν ορεσσιν,
Χερσιν εχων ῥοπαλον παγχαλκεον αιεν ααγες.

"Poſteà, *Orionem ingentem* aſpexi,
Feras unà cogentem per aſphodelum pratum,
Quas *Ipſe* occiderat in montibus deſertis;
Manibus tenentem clavam prorsùs æneam, ſemper infractam."

Annon facilè derivari poteſt *Ωριων* à *Chaldæo Uriah* ("Lux Domini"), quæ in Verſione Græcâ *Septuagint*. in caſu obliquo, redditur Ουριαν, 2 *Sam*. xi. 6.?

(*k*) Ingentis hujuſce tyranni nomen proprium fuiſſe נין *Nin*, "filius," cui per odium et ludibrium cognomen indiderunt *Shemitæ* ſubjugati, את נמרד *Æth Nimrod*, "Archirebellem," confirmatur ex *Trogo Pompeio*, ſeu *Juſtini* Lib. I. initio.

"Primus omnium *Ninus*, Rex Aſſyriorum, veterem et quaſi *avitum* gentibus morem, [i. e. *Regimen Patriarchum*,] novâ imperii cupiditate mutavit. Hic primus intulit bella *finitimis*, et rudes adhùc ad reſiſtendum populos, terminos uſque *Libyæ*, perdomuit."—"Domitis igitùr proximis, cum acceſſione virium fortior ad alias tranſiret; et proxima quæque victoria inſtrumentum ſequentis eſſet, totius *Orientis* populos ſubegit. Poſtremùm illi bellum cum *Zoroaſtre* Rege *Bactrianorum* fuit (qui primus dicitur et *Sheba*, adeóque *Pelegi* co-ævus, quintus nimirùm à *Noab*, non tertius, ut hactenùs à Chronologis malè ſupponitur.

179. Ingruenti huic idololatriæ fortitèr ſe oppoſuit *Job*, ſeptimus vel octavus à *Noah*. (Vide *Abulfaragi*, p. 13.)

> "Si aſpicerem *lucem** [*Solis*] ubi ſplendebat, *אור·
> Aut *Lunam* gloriâ incedentem;
> Si cor meum in occulto ſeductum eſſet,
> Aut manus mea ab ore oſcularetur;
> Etiam hoc foret *crimen judiciale*,
> Ita enim mentitus eſſem Deo Supremo." *Job* xxxi. 26.

180. Sed, *damnoſa quid non imminuit dies?*—Sublimem et ſinceram Patriarcharum *Chaldæorum* atque *Hebræorum* Theologiam—"Spiritum Supremum in *ſpiritu* atque *veritate* ritè venerantium," tandem corrupit et inundavit *Sabianiſmus*, ſeu praviſſima idololatria aſtrorum; "*fingebant enim idololatræ*," ut benè notat *Newtonus*, *Princip*. p. 529.—"*Solem*, *Lunam* et *Aſtra*, *Animas Hominum*, et alias *Mundi partes*, eſſe *partes* Dei Summi; et ideò *colendas*, ſed falsò.—Atque ex cunctis Philoſophis Mythologíſque *Gentilibus*, eheu!

> "*Apparent rari nantes in gurgite vaſto*,"

qui de Deo ritè ſentiebant.

181. Operam Philoſophis noſtris *Britannis* haùd ingratam me impenſurum ſpero, ſi curſìm ex *Græcis Romaníſque* Mythologis, miram et plorandam ſanè cœcutiam atque vecordiam paucis exemplis illuſtrem, de *Deo materiali* per vocem Αιθηρ, vel *Æther*, ab utríſque intellecto, turpiſſimâ Priſcæ Theologiæ depravatione.

Tragœdorum Græcorum antiquiſſimus, *Æſchylus*, *Ætherem*, eſſe *Deum* apertè poſuit.

---

dicitur *Artes Magicas* [ſeu diſciplinas *Magorum*] inveniſſe, et *mundi principia*, *ſiderúmque motus ſpectâſſe*.) Hoc occiſo, et ipſe deceſſit; relicto impubere adhùc filio *Ninya* et uxore *Semiramide*—"

Hìc, *Juſtinus*, more *Græciæ* mendacis, vera falſis immiſcuit. *Nimrodum* probè refert, finitimos invadentem, atque interficientem ultimò, paulò antè obitum, *Zoroaſtrem*, (forsàn *Peleg* ipſum, in Montibus *Ararat*,) Archimagum, peritiſſimúmque Aſtronomum. Sed errore planè ridiculo, ob inſcitiam Linguarum Orientalium, *Trogus* tranſtulit *urbem* Νινευ à *Nimrod* conditam, ſeu *Niniven*, in *filium* "*Ninyam!*"—Et de *Semiramide* fabulatur! Vera enìm Semiramis, quinque tantùm generationibus, teſte *Herodoto*, anteceſſit *Nitocrem*, inſignem Reginam, quæ Babyloni exſtruendo atque ornando ultimam manum impoſuit; illa autèm videtur fuiſſe magni *Nebuchadnezzari* uxor, de quo *Daniel*, v. 10.

Secundùm emendationem noſtram *Chronologiæ Sacræ*, incidit diſtributio terrarum habitabilium inter Colonias Noachidarum, "in diebus *Peleg*," (i. e. "*Diviſionis*,") ubi floruit ille *Ninus* Archirebellis, anno 541 circitèr, poſt diluvium; ſeu anno 2615 antè Æram Chriſtianam: floruit autèm illa *Semiramis* quæ mœnia *Babylonis* conſtruxit, 166 annis ſaltèm antè obitum *Nebuchadnezzar*, ſive 166 + 561 = 727 antè Æram Chriſtianam. *Semiramis* proindè pro uxore *Nabonaſſari*, (cujus æra incepit 747 annis antè Chriſt.) jure haberi poteſt.

ΖΕΥΣ εστιν ΑΙΘΗΡ, Ζευς δε Γη, Ζευς δ᾽ Ουρανος, Ζευς τοι Παντα.

" Jovis eſt ÆTHER; *Jovis* item *Terra*, et *Jovis Cælum*, *Jovis* deníque Omnia."

182. Audi quoque *Hippocratis* commentum, *De Princip.* § 1.

Δοκει δε μοι ὁ καλεομεν ΘΕΡΜΟΝ, *αθανατον τε ειναι και νοεειν παντα, και ὁρην και ακουειν και ειδεναι παντα, και τα οντα και τα εσομενα· τουτο ουν, το πλειστον, ὁτε εταραχθη παντα εξεχωρησεν ες των ανωτατω περιφορην· και ονομηναι μοι αυτο δοκεουσιν οἱ παλαιοι* ΑΙΘΕΡΑ.

Ἡ δευτερα μοιρα, ἡ κατωθεν αυτη, καλεεται μεν ΓΗ, *ψυχρον και ξηρον και πολυ κινουν· και εν τουτεῳ ενη δη πολυ του Θερμου. Ἡ δε τριτη μοιρα, ἡ και του* ΗΕΡΟΣ *χωριον ειληφε, θερμον τι και ὑγρον εον. Ἡ δε τεταρτη, ἡ του εγγυτατω προς τῃ Γῃ, ὑγροτατον τε και παχυτατον.*

" Mihi quidĕm videtur Id quod CALIDUM vocamus (Gallicè CALORÍQUE) eſſe *immortale*; atque omnia *percipere* et *videre* et *audire*, omniáque *ſcire* tam *præſentia* quam *futura*. Hoc igitùr, maximâ ex parte, omnibus perturbatis [*chaoticè*], exceſſit in orbem altiſſimum; ídque nominâſſe mihi videntur Priſci ÆTHEREM."

" Secunda pars, eáque inferior, vocatur TERRA. Eſtque *frigidum* et *ſiccum* et *permobile*; in hâc quoque ineſt *calidi* multum. Tertia autèm pars, quæ *aëris* regionem ſortiebatur, eſt quoddam *calidum* ſimùl et *humidum*. Pars verò quarta, terræ proxima, eſt *humidiſſimum* ſimùl et *denſiſſimum*."

183. Et convenientèr *Virgilius*, *Æn.* VI. 724.

> " Principio *Cælum* et *Terras*, *Campóſque liquentes*,
> Lucentémque globum *Lunæ* Titaniáque *Aſtra*,
> SPIRITUS intùs alit; totámque, infuſa per artus,
> MENS agitat molem, et magno ſe corpore miſcet."

Altiùs hìc ſonant " *Spiritus* et *Mens*;"—ſed ſenſu prorsùs *materiali*, intelligenda; ut ex ſequentibus patebit, *Georg.* II. 340.

> " Cum primum *lucem pecudes* hausêre, *virûmque*
> Ferrea progenies duris caput extulit arvis,
> Immiſſaéque *Feræ* ſylvis, et *Sidera* cœlo."

184. Rationem reddente *Ovidio*, *Metam.* I. 75.

> " Neu regio foret ulla *ſuis animantibus* orba,
> *Aſtra* tenent *Cæleſte Solum*, *Formaéque Deorum*,
> *Terra Feras* cepit."———

185. Pleniùs autèm et luculentiùs, *Virgilio*, *Georg*. IV. 219.

> "His quidam ſignis, atque hæc exempla ſecuti,
> Eſſe *apibus* partem DIVINÆ MENTIS, et HAUSTUS
> ÆTHEREOS, dixêre; DEUM nàmque ire per omnes
> *Terráſque*, tractúſque *Maris*, *Cælúmque* profundum.
> HÌNC, *Pecudes*, *Armenta*, *Viros*, genus omne *Ferarum*,
> Quemque ſibi tenues *naſcentem* arceſſere vitas:
> Scilicet HÙC reddi deinde, ac reſoluta referri
> Omnia; nec *Morti eſſe locum*; ſed *Viva* volare
> *Sideris* in numerum, atque alto ſuccedere *Cælo*."

186. Et apertiſſimè quidem, *Georg*. II. 323.

> "Tùm PATER OMNIPOTENS, fœcundis imbribus ÆTHER,
> *Conjugis* in gremium lætæ deſcendit, et omnes
> Magnus alit, magno commixtus corpore, *fœtus*."

Hìc procùl dubio, "*Pater Omnipotens*" et *Æther*, pro ſynonymis habentur à *Virgilio*; ſicùt à cæteris paſsìm Poetis *Latinis*,—JUPITER et ÆTHER.

187. Paritèr *Horatius*, *Od*. I. xxii. 19. et I. i. 25.

> "Quod latus mundi, *Nebulæ*, MALUSQUE
> JUPITER *urget*."
>
> "———Manet ſub JOVE FRIGIDO
> Venator."———

188. Paritérque *Lucanus*:

> "JUPITER eſt quodcunque *vides*, quocunque *moveris*."

189. Atque identitatem *Mentis Divinæ* et *Humanæ* apertiſſimè tradiderunt Philoſophi, referente *Cicerone*:—"Sive *anima* ſive *ignis* ſit animus (*humanus*) eum jurarem eſſe *divinum*."—"Et quidem ſi *Deus* aut *anima* aut *ignis* eſt, idem eſt animus hominis:"—"Ergò *animus*, ut ego dico, *divinus* eſt; ut *Euripides* audet dicere, *Deus*."

Et ex hominum motu libero argumentum fabricârunt pro animi æternitate: ut docet quoque *Cicero*:—"Quòd *ſe ipſum moveat*, quis eſt qui hanc naturam *animis* eſſe tributam neget? *Inanimum* enìm eſt omne quod *pulſu* agitatur *externo*; quod autèm eſt *animal*, id *motu* cietur *interiore* et *ſuo*.—Sentit igitùr animus *ſe moveri*; quod cum ſentit, illud ſentit, ſe *vi ſuâ*, non alienâ, *moveri*; nec *accidere poſſe* ut ipſe unquam à ſe deſeratur; ex quo efficitur *æternitas*." (Vid. § 193.)—"Ex DIVINITATE *animos* hauſtos habemus."

190. Talia ſunt dogmata "*Inſanientis Sapientiæ*"—CREATOREM cum *creatis*, *homines* cum *beſtiis*, *animantia* cum *inanimatis*, confundentis; SPIRITUI SUPREMO, exiſtentiam *materialem* impiè tribuentis, *immortalitatem* innatam *animorum hominum pecudúmque* falsò prædicantis; DEUM OPTIMUM MAXIMUM, (cujus eſt "*dare* et *adimere*," vitam commodare et vitam perdere, creare atque annihilare, ſolo ſui ipſius numine, et liberrimo arbitrio, nullo cogente *Fato* ſeu *Neceſſitate cæcâ*,) ingratè prætermittentis et turpitèr dehoneſtantis!

191. Optimè igitùr et acutiſſimè *Paulus* de Philoſophis Gentilibus:—*Rom.* i. 22. (*l*).

Φασκοντες ειναι ΣΟΦΟΙ, ΕΜΩΡΑΝΘΗΣΑΝ.

"Autumantes ſe eſſe SAPIENTES, facti ſunt STULTI."

(*l*) Quantum autèm inſipuerunt, et quàm miſerè cæcutierunt, vel ævo *Auguſti* literarum culturâ celebrato, Philoſophi *Romani*, uno alteróve exemplo oſtendere libet:

Gravem illam, ſublimem et magnificam, DEI SUMMI deſcriptionem *Horatianam* quis non miratur humaniorum litterarum ſcientiâ vel levitèr imbutus? Od. III. 4, 42.

——"Scimus ut impios
Titanas, immanémque turmam
Fulmine ſuſtulerit caduco,
Qui *terram inertem*, qui *mare* temperat
Ventoſum; et *urbes*, *regnáque triſtia*,
Divóſque, *mortaléſque* turbas,
Imperio regit UNUS *æquo*."

Petuntur hæc ſanè ex *Philoſophiâ primâ*, et manifeſtò reſpiciunt *Sodomi* atque *Gomorræ* excidium, *Gen.* xix. 24.; aut *Gigantes* forsàn, *Gen.* vi. 4. Diluvio peremptos. Sed quàm indigna proximè ſequuntur?

"*Magnum* illa *terrorem* intulerat JOVI,
Fidens *Juventus* horrida brachiis,
Fratréſque tendentes opaco
Pelion impoſuiſſe Olympo!"

Quàm incongrua, quàm abſurda, quàm impia cadit concluſio?—Quinque mortales juvenes nempè "*terrorem intuliſſe* DEO SUMMO," ne cœlum ſcandant ipſúmque à ſolio detrudant!!!—Nónne in hoc caſu Poetarum *Ethicorum* princeps (ut fertur *Horatius*), factus eſt ſtultiſſimus?

Et quis feret vel ipſum *Ciceronem* praviſſimè philoſophantem de *perjurio*, quaſi à NUMINE SUMMO prorsùs neglecto atque impunito? *De Officiis*, III. § 28.

"Quid eſt igitùr, dixerit quis, in *jurejurando*? *Num iratum timemus* JOVEM? At hoc quidem COMMUNE EST OMNIUM PHILOSOPHORUM (non eorum modò, qui *Deum nihil habere ipſum negotii, et nihil exhibere alteri* [ut *Epicureorum*]; ſed eorum etiàm qui, *Deum ſemper agere aliquid et moliri* volunt [ut *Stoicorum*, *Peripateticorum*, &c.]) NUNQUAM NEC IRASCI DEUM NEC NOCERE."

Nónne Sophiſtæ tales, (ſicut Cicero *Epicureos* incuſat),

"DEUM *nomine ponunt*, *re tollunt*?"

Atque eò ſtultitiæ et vecordiæ progreſſi ſunt, ut Numen Summum, paſsim, per vocem TO ΘEION deſignârunt, cujus ſignificatio communis apùd Veteres, fuit "*Sulphur Sacrum,*" Anglicè "*Lightning.*"

Sic *Lucas* Caſum *Sodomi* referens, xvii. 29. ex *Geneſi*, xix. 24.

> Εβρεξε πυρ και θειον, (Κυριος) απ' ουρανου.
>
> "Depluit (Dominus) *ignem* et *ſulphur* de cœlo."

Et phraſin claſſicam, θειου πληγο, exponit *Heſychius*, του κεραυνιου πυρος λεγει· οδμην γαρ εχει ὁ κεραυνος θειου,—"*Ignem fulmineum* vult; habet enìm *fulmen ſulphuris odorem.*" Appellatur autèm Θειον—το εκ Θεου αφιγμενον, utpote "ex Deo *proventum.*"

192. Notatu digniſſimum eſt quòd unico tantùm Evangelii loco, uſurpatur TO ΘEION in ſenſu philoſophico, pro Divinitate ſeu Numine Summo, ídque à *Paulo*, hallucinationem *Stoicorum* atque *Epicureorum* Athenis perſtringente:—

"Minimè reputare debemus, auro vel argento, vel lapidi, ſculpturæ nimirùm artis ſolertiaéque *humanæ* (TO ΘEION) Divinitatem eſſe ſimilem."—*Acts* xvii. 29.

Omnibus aliis in locis vox Θειον in ſenſu communi adhibetur, pro ſulphure ſacro.

193. Quanto acumine refellit *Paulus* dogmata *Ariſtotelica* de *motu neceſſario*, ſeu *mechanico*:—

Τις ἡ της κινησεως αρχη τῃ ψυχῃ; δηλον δε ὡσπερ εν τῳ ὁλῳ ΘΕΟΣ, και παν εκεινῳ· Κινει γαρ πως τα παντα το εν ἡμιν ΘΕΙΟΝ.—Ἑν τι των αδυνατων το ὑπαρχειν ψυχῃ κινησιν.

"Quid eſt principium motûs *animo?* Patet ſanè, ſicùt in toto Deus, ità quoque Omne in Illo: *Movet* enìm quodammodo omnia *in nobis* Divinitas."—"Ex impoſſibilibus unum eſt quoddam, *animo competere motum.*"

Contrà aſſerit Apoſtolus, *motum* hominum *voluntarium*, et tamèn à Deo oriundum:—Εν ΑΥΤῼ (ΘΕῼ) γαρ ζωμεν και κινουμεθα και εσμεν—"In Illo (Deo) enìm *vivimus* et *noſmet-movemus* et *ſumus.*"—Nam κινουμεθα vox media eſt.—Quâ ratione autèm *noſmet ſponte movemus*, mortalibus ſcire non fas eſt; à Deo hæc facultas conceditur, ſed more nobis prorſùs incognito, et nunquàm forsàn cognoſcendo. Vide § 189.

194. Sufficiant hæc de *Æthere*, ſeu De *Deo Sulphureo* et *Materiali* Sophiſtarum Veterum, hoc noſtro *ſæculo rationis* (ut dicitur) infauſto, eheu! renaſcentium. Annon poeta noſter Ethicus quoque, *Pope*, inſcius forsàn, in hunc turpiſſimum

errorem *Stoicorum* et *Peripateticorum*, à *Carteſio* et *Leibnitzio* malè recoctum, lapſus eſſe videur, ubi *Deum* hallucinans *Animam Mundi* refinxerit?

> "All are but *parts* of one ſtupendous *whole*,
> Whoſe *body* NATURE is, and GOD the *ſoul*."

Quanto meliùs fuiſſet, ſi à diſquiſitionibus ſpinoſiſſimis et *periculoſæ* plenis *aleæ*, quæ poëtis (qui plerumque in hiſce ſtudiis ſeverioribus parùm ſunt verſati) minimè conveniunt, prorsùs abſtinuiſſet, dogmatis illius ſani à ſe-ipſo alibì prolati ritè memor!

> "A *little learning* is a *dangerous* thing;
> *Drink deep*, or *taſte not* the PIERIAN SPRING."

Vide § 207.

195. Regerent hìc forsàn Poetæ, *Popiique* fautores, "nònne ipſe *Newtonus* pariter quoque peccat, ubi de *Senſorio* DEI differit in ſublimi argumento de exiſtentiâ ENTIS SUPREMI ex operibus Naturæ cauſiſque finalibus patefacti?" *Optics*, p. 345.

"Does it not appear from *phænomena* that there is a BEING *incorporeal*, *living*, *intelligent*, *omnipreſent*, who in infinite ſpace (as it were in *his Senſory*) ſees the *things themſelves* intimately, and thoroughly perceives them, and comprehends them wholly by their immediate preſence to HIMSELF: of which things the *images* only, carried through the organs of ſenſe into our *little ſenſoriums*, are there ſeen and beheld by *that* which in us perceives and thinks?"

But how cautiouſly, how correctly does *Newton* guard againſt miſrepreſentation of his meaning, in a ſubſequent paſſage, p. 379.

"Such a wonderful *uniformity* in the *planetary* ſyſtem—the *uniformity* in the bodies of *animals*—the *firſt contrivance* of theſe very artificial parts—the eyes, ears, brain, muſcles, &c. and other organs of ſenſe and motion; and the *inſtinct* of brutes and inſects, can be the *effect* of nothing elſe than the *wiſdom* and *ſkill* of a *Powerful*, *Everliving* AGENT; who, being in all places, is more able by *his will* to move the bodies within his boundleſs uniform *ſenſorium*, and thereby to form and reform the parts of the Univerſe, than we are by *our will* to move the parts of our own bodies.

"And yet, we are not to conſider "*the World* as the *body* of GOD, or the ſeveral parts thereof as the *ſoul* of GOD:"—HE is an UNIFORM BEING, void of organs, members or parts; and *They* are his *creatures*, ſubordinate to HIM, and ſubſervient to *his will*: and HE is no more *the ſoul of them*, than the *ſoul of man* is the ſoul of the *ſpecies* of things carried through the organs of ſenſe into the place of it's ſenſorium; where *It* perceives them by means of it's immediate preſence, without the intervention of any *third* thing.

[N. B.] "The *Organs of Senſe* are not for *enabling the ſoul to perceive* the ſpecies of things in it's ſenſorium; but only for *conveying* them thither: and God has no need of ſuch organs; He being every where preſent to the things themſelves."

DE ENTE SUPREMO.

196. Audiamus *Newtonum* quoque, tunc temporis grandævum, graviſſimè et ornatiſſimè de hoc myſterio maximè incomprehenſibili differentem, in *Scholio generali Principiorum*, nunquàm ſatìs laudando.

"Hic omnia regit, non ut *Anima Mundi*, ſed ut *Univerſorum Dominus.* Et proptèr dominium ſuum Παντοκρατωρ (id eſt, *Imperator Univerſalis*), dici ſolet: Nam *Deus* eſt vox relativa, et ad *ſervos* refertur; et *Deitas* eſt *Dominatio Dei*, non in corpus proprium, (utì ſentiunt quibus Deus eſt *anima mundi*), ſed in ſervos.—Vox *Deus* paſsìm ſignificat *Dominum* (*m*). *Dominatio* Entis Spiritualis, Deum conſtituit, vera verum, ſumma ſummum, ficta fictum. Et ex *dominatione*

De Nominibus Dei Primitivis.

(*m*) *Newtonus*, "*Pocock* noſtrum ſecutus, vocem Latinam *Dei* deducit à voce Arabicâ *Du*, (et in obliquo caſu, *Di*), quæ *Dominum* ſignificat,"—At minùs rectè, ut videtur. Et forsàn, haùd importunum erit, pro munere meo *profeſſorio*, quamvis *emerito*, de ſacroſanctis Dei nominibus primitivis, cautè, verecundè et circumſpectè ex rationibus *etymologycis*, ſanioribus magíſque ſcientificis, forsàn, quam hactenùs per *Lexica* quævis *Orientalia* vel *Inſtitutiones Hebræas* prolatis, brevitèr differere.

'ΑΓΙΑΣΘΗΤΩ ΤΟ ΟΝΟΜΑ ΣΟΥ.

"*Sanctificetur Nomen Tuum.*"

Innuit *Pocock*, nimio Arabiſmi ſtudio illectus, Linguam *Arabicam* eſſe *primævam.*—Cæterùm omnium matrem eſſe *Hebræam*, ſatìs conſtabit ex nominibus propriis, *Virorum*, *Regionum*, *Fluviorum* et *Urbium*, tàm antè diluvium quàm per quinque generationes poſt; quæ omnia ſunt purè *Hebræa*, et plurima ex cæteris dialectis primitivis exulant. Et neceſſariò, quidèm, ſi Lingua *Noachidarum* erat eadem ac *Antediluvianorum*; et ſi "*Totus Mundus fuit unius linguæ uniúſque dialecti*" uſque ad *Diſperſionem Babelitican* et *Linguarum Confuſionem.* *Gen.* xi. 1.

Vox Latina *Deus* rectiùs deduci poteſt immediatè à Græcâ Δευς, Doricè pro Ζευς,—(*Heſych.*)—tranſmutatione literarum *cognatarum* per omnes gentes uſitatiſſimâ; atque hæc iterùm à voce Phœniciâ Ιευω, qualem pronunciabant nomen *Dei* tetragrammatum apud *Hebræos* ſanctiſſimum, יהוה, *Ieuhōh*; (*Scaliger*, ex *Sanchoniathone*, De *Emend.* Temp. *Fragm.* Not. p. 37.) ſimili tranſmutatione, et pro terminatione Hebræâ וה, *ōh*, ſubſtituendo quóque terminationem Σ, à Linguâ *Medâ*, ſeu Perſicâ primitivâ, mutuatam; (*Herodotus*, I.) Antiquitùs autèm, ab Oraculo *Clarii Apollinis*, pronunciabatur Ιαω, quaſi *Iahōh*, ſciſcitantibus "*quis Deorum habendus ſit qui vocatur* Ιαω?" (*Macrob.* I. 18.)—Undè Priſcorum Græcorum Ζας et Ζαν—Ζανος, et Ionicè Ζην—Ζηνος—(*Heſych.*) et Priſcorum *dominatione verâ* ſequitur **Deum verum** eſſe *vivum*, *intelligentem* et *potentem*: ex reliquis perfectionibus *ſummum* eſſe vel *ſummè perfectum*.

"Deus eſt Unus et Idem Deus, *ſemper et ubique*. *Omnipreſens* eſt, non per *virtutem* ſolam, ſed etiàm per *ſubſtantiam*: nam virtus ſine ſubſtantiâ ſubſiſtere non poteſt. In Ipſo *continentur* et *moventur* Univerſa, ſed ſine mutuâ paſſione: Deus nihil patitur ex *corporum motibus*; illa nullam ſentiunt reſiſtentiam ex *Omnipræſentiâ* Dei.

"Deum

Priſcorum Italorum, *Ianus*, Deorum antiquiſſimus, qui "*Saliorum* quóque antiquiſſimis carminibus *Deorum Deus* canitur." *Macrob.*

Hanc vocem Ιαω contractiùs pronunciabatur Ιῶ, quam malè confundunt omnes ferè editores Græci atque Latini, cum interjectione O! ut *Euripid. Bacchæ*, 583.

> Ιω, Ιω, Δεσποτα, Δεσποτα.
>
> "*Io, Io, Domine, Domine.*"

—pro Ιω Βακχε, "*Io Bacche*," *Hor*. Sat. I. 3. 7.

Itémque *Virgilius*, archaiſmi obſervantiſſimus, *Æn*. 10. 17.

> "*Io-pater, Io*—Hominum Divúmque *æterna poteſtas*."

—quæ vulgò *Jupiter*, pronunciatur, at in caſibus obliquis, *Io-vis*, *Io-vi*, &c. verum ſuum etymon declarat. Vocem Ιῶ interpretatur Heſychius, *ενι*, *uno*; et confirmatur ex *Homeri* Ionicâ, in phraſi Ιῶ ἠματι, *Iliad*, 6. 422. interpretante Scholiâ, εν μιᾳ ἡμερᾳ, "*uno die*." Optimè autèm conſentit Sacra Scriptura, *Deut*. vi. 4. et *Mark* xii. 29.

> "*Audi Iſrael, Dominus Deus noſter Dominus* unus *eſt*."

Et notatu digniſſimum eſt quod nunquàm ab Hebræis יהוה, Ιαω, nunquàm à Græcis, *Ζας* aut *Ζευς*, *pluralitèr* uſurpantur, utpote *unitatem Dei* deſignantes.

> ʽΕις Ζευς, ἑις ʽΑδης, ἑις ʽΗλιος, ἑις Διονυσος.
>
> "*Unus Jovis, unus Pluto, unus Sol, unus Bacchus.*"

Ut refert *Orpheus*, citante *Macrobio*, qui cum *Oraculo Clario* conſentit.

Et vox ipſa יהוה ultimò deduci videtur à radice primævâ יה *Iah*; quæ nomen Dei antiquiſſimum apud Hebræos fuit, (ſicùt apud Italos, *Ianus*); et cujus propria ſignificatio optimè forsàn ab *Homero* aſſervatur in phraſi Ια γηρυς, *Iliad* 4. 437. interpretante Scholio, μια και αυτη φωνη, "*una eadémque lingua*."—Et optimè reſpondet hæc ſignificatio, *Iſa*. xxvi. 4. ubi יה et יהוה ſimùl occurrunt:

> "*Fidite Domino uſque in perpetuum,*
> Quia in (*Jah, Iahoh*) *uno eodémque Domino* eſt *Petra Sæculorum*."

Egregiè igitùr deſignat radix יה Divinam *immutabilitatem* et *æternitatem*. Confer *Malach*. iii. 6. *Jam*. i. 17. *John* xiv. 1. *Heb*. xiii. 8.

2. A voce Hebræâ, די *Di*, quæ propriè ſignificat *Sufficientiam*, *Copiam*, *Opes*, et undè epitheton divinum שדי *Sadi*,—"*cujus* eſt *ſufficientia*," *Gen*. xvii. 1. et 2 *Cor*. iii. 5. quaſi ab radice exſurgunt Arābicâ, *Sadi*; Græca Δις, Δι-ος, quæ *Deum Summum* deſignabat; cum voce Ζευς ſynonomans, et ſibi poſteà caſus obliquos Δι-ος, Δι-ι, Δι-α, aſſumens; et Latina *Dis*, *Di-tis*, Deum Infernum, *Plutonem*, turpiore Theologiæ Primitivæ depravatione; et indè quoque, *Dii Manes*, ſeu *Dæmones*, mortuorum umbræ, quas praviſſimâ ſuperſtitione colebant gentes.—Hìnc *Jupiter* appellatur *Dis-piter* ab Horatio, diſtinctionis gratiâ, non *Dieſpiter*, ut imperitè editores.

3. A voce Hebræâ אלוה *Ælōh*, quæ propriè ὁ Δυναςης vel Δεσποτης, "*Dominus*," vel "*Imperator*" ſignificat, 1 *Tim*. vi. 15. vel *Jude* iv. Atque Deo Summo antiquitùs à gentibus omnibus tributa

"Deum Summum *neceſſariò exiſtere*, in confeſſo eſt. Et eâdem neceſſitate *ſemper* eſt et ubique. Unde etiàm totus eſt SUI-SIMILIS: Totus oculus, totus auris, totus cerebrum, totus brachium, totus vis ſentiendi, intelligendi et agendi; ſed more minimè humano, *more nobis prorsùs incognito*. Ut cæcus non habet ideam colorum, ſic nos ideam non habemus modorum quibus DEUS *Sapientiſſimus* ſentit et intelligit omnia.

"*Corpore omni* et *figurâ corporeâ* prorsùs deſtituitur; ideóque videri non poteſt, nec tangi, nec ſub ſpecie rei alicujus corporeæ coli debet. Ideas habemus *attributorum* ejus, ſed quid ſit alicujus rei *ſubſtantia* minimè cognoſcimus. Videmus tantùm corporum figuras et colores, audimus tantùm ſonos, tangimus tantùm ſuperficies externas, olfacimus odores ſolos, et guſtamus ſapores; intimas ſubſtantias nullo ſenſu, nullâ actione reflexâ cognoſcimus; et multò minus ideam habemus ſubſtantiæ Dei.

"HUNC *cognoſcimus* ſolummodò per *proprietates* ejus et *attributa*, et per *ſapientiſſimas* et *optimas* rerum *ſtructuras*, et *cauſas finales*; et *admiramur* ob *perfec-*

---

tributa eſt, ritè deducitur variabilis *Allah*, *Ullah* vel *Alo*, quæ vel hodiè cunctas ferè regiones Orientales peragravit; atque ipſa אלוה *Ælōh*, pari analogiâ, ex nomine Dei longè antiquiſſimo omnium, אל *Æl*, facilè derivatur; quæ abſtractè ſignificat ἡ δυναμις, "*Potentia*," concretè verò, ὁ Δυνατος, "*Potens*."—Et notio Dei primaria et omnium ſimpliciſſima et uſitatiſſima erat *Potentiæ*, ſive beneficæ ſive maleficæ. Vid. *Gen.* xxxi. 29. ubi minitatur *Laban Jacobo* genero ſuo profugo—"Eſt *potentiæ* manus meæ te *nocere*,"—לאל. Vide quoque 1 *Sam.* xvii. 26. et 2 *Kings*, v. 7. et *Rom.* i. 20. ubì *Paulus* pro parallelis recenſet DEI SUMMI Δυναμις και Θειοτης, "*Potentia* et *Deitas*,"—quem probè ſequitur *Newtonus*—"*Deitas eſt Dominatio*."

4. Gliſſante verò Idololatriâ, malè haberi incœpit vox אל *Æl*, pro *Sole*, numinibúſque fictis Mythologiæ Veterum; unde Solis epitheton, Αελ-ιος, Ηλ-ιος, vel ʽΗλ-ιος, in diverſis Græciæ (in Theologiâ mendaciſſimæ) dialectis. Sic plorat *Phaëthon*, fulmine ictus. *Euripid.*

Ω χρυσοφεγγες ʽΗλι', ὡς μ' απωλεσας;
ʽΟθεν Σ' Απολλων' εμφανως κλησει βροτος.

"O aureâ tædâ prædite *Sol*, quomodo me *perdidiſti!*
Undè Te *Perdentem* clarè vocant mortales."

Atque inde pluralitèr uſurpari cœperunt אלים et אלהים *Ælim* et *Ælohim*, pro *Angelis*, *Divis*, *Judicibus*, *Heroibus*, &c.—Unde ad retinendam præſtantiam DEI SUMMI, cum epithetis variis uſi ſunt Hebræi Prophetæ, אל אלים, *Æl Ælim*, *Dan.* xi. 36. "*Deus Divorum*;"—אל אלהים, *Æl Ælohim*, "*Deus Deorum*;"—יהוה אלהים, *Iahoh Ælohim*, "*Dominus Deorum*;"—et אל אלהים יהוה *Æl Ælohim Iahōh*, ſicùt pleniſſimè deſcribitur Nomen Dei ſacroſanctum, *Joſh.* xxii. 22. *Pſ.* l. 1. "*Dominus Deus Deorum*."

5. Atque hinc, ellipſi frequentiſſimâ אלהים *Ælohim*, licet vox pluralis, uſurpatur ſingularitèr: ut *Gen.* i. 1. אלהים ברא *Deus creavit*; ſubaudito vel יהים *Iahōh*, vel אל *Æl*, vel utroque. Atque hoc pacto Scripturis Hebræis grammaticè amovetur *ſolæciſmus* iſte, qui mirè torſit interpretes, atque varia abſurda, imò impia, commenta, *Rabbinornm* et *Myſticorum* peperit; ut videre licet in Lexicis Scriptíſque *Kimchi*, *Buxtorf*, *Parkhurſt*, &c. &c. &c.

6. Ex his igitùr nominibus Dei ſacroſanctis, omnium vetuſtiſſimis atque maximè radicalibus, אל *Æl*, et יה *Iah*, utpote ſimpliciſſimis, (unde deducuntur Dei nomina propria in omnibus Linguis primitivis, necnon in dialectis, tàm Orientalibus quàm Occidentalibus;) vel ex his ſolis non inficiabuntur Etymologi æqui et periti, omnium matrem, ipſámque primævam linguam, fuiſſe *Hebræam*. Sed hæc paucis exponi non poſſunt.

tiones; veneramur autèm et *colimus* ob *dominium*: colimus enìm ut ſervi, et Deus ſine *dominio*, *providentiâ* et *cauſis finalibus*, nihil aliud eſt quàm *Fatum* et *Natura*: à cæcâ *Neceſſitate Metaphyſicâ*, quæ utique eadem eſt ſemper et ubique, nulla oritur *rerum variatio*: tota rerum conditarum *diverſitas* ab *ideis* et *voluntate* Entis necessariò Existentis ſolummodò oriri *potuit*.

“ Dicitur autèm Deus per allegoriam, *videre*, *audire*, *loqui*, *ridere*, *amare*, *odio habere*, *cupere*, *dare*, *accipere*, *gaudere*, *iraſci*, *pugnare*, *fabricare*, *condere*, *conſtruere*: nam ſermo omnis de Deo, à *rebus humanis* per ſimilitudinem aliquam deſumitur, non perfectam quidem, ſed *aliqualem* tamen.

“ Et hæc de Deo; de quo utique ex *phænominis* diſſerere ad Philosophiam Naturalem pertinet.”

197. In hoc *Scholio generali* ſummè *Theologico*, objiciunt Metaphyſici *Leibnitziani*, Newtonum denegare exiſtentiam *ſpatii* et *durationis* abſolutam, ubì aſſerit, p. 528:

“ Deus non eſt *æternitas* et *infinitas*, ſed *æternus* et *infinitus*; non eſt *duratio* et *ſpatium*, ſed *durat* et *adeſt*: durat *ſemper* et adeſt *ubique*; et *exiſtendo ſemper et ubique*, *durationem et ſpatium*, [*æternitatem et infinitatem*] *conſtituit*.”

198. Cæterùm reſponſio facilis: Si non exiſteret Deus, quo pacto ſpatium et duratio ab ullo *Ente intelligente cernerentur?* Nam, ut ipſi loquuntur Metaphyſici, *de non apparentibus et de non exiſtentibus eadem eſt ratio.*—*Newtono* adſtipulatur ſublimis illa Theologia Patriarcharum et Evangeliſtarum: docuit enìm *Orpheus*, *Moſem* ſecutus, quod “ *Deus eſt Lux*—*Lucem habitans inacceſſibilem*,” et confirmat *Paulus*, in celeberrimâ illâ Dei Summi deſcriptione, omni laude longè longiſſiméque majore, quàm potiſſimùm ſecutus atque interpretatus eſt *Newtonus*: 1 *Tim.* vi. 15.

> ʽΟ ΜΑΚΑΡΙΟΣ ΚΑΙ ΜΟΝΟΣ ΔΥΝΑΣΤΗΣ,
> ʽΟ ΒΑΣΙΛΕΥΣ ΤΩΝ ΒΑΣΙΛΕΥΟΝΤΩΝ,
> ΚΑΙ ΚΥΡΙΩΣ ΤΩΝ ΚΥΡΙΟΝΤΩΝ·
> ʽΟ *μονος εχων αθανασιαν*·
> *Φως οικων απροσιτον*·
> ʽΟ*ν ειδεν ουδείς ἀνθρωπων, ουδε ιδειν δυναται*·
> ʽΩ ΤΙΜΗ ΚΑΙ ΚΡΑΤΟΣ ΑΙΩΝΙΟΝ. ΑΜΗΝ.

> “ Felix atque Solus Imperator,
> Rex Regentium et Dominus Dominantium;
> *Solus habens immortalitatem*,
> *Lucem habitans inacceſſibilem*,
> *Quem vidit nemo mortalium, nec videre poteſt*:
> Cui Honos et Dominium Æternum. Amen.”

199. Confirmat quóque *Johannes:*

‘Ο ΘΕΟΣ *φως εστι, και Σκοτια εν Αυτῳ ουκ εστιν ουδεμια.*

“ Deus *Lux* eſt, atque in Illo haudquaquàm eſt *caligo.*”

Πνευμα ‘Ο ΘΕΟΣ—‘Ο ΘΕΟΣ *Αγαπη εστι.*

“ *Spiritus,* Deus ”—“ Deus eſt *Charitas.*”

At ſi prædicantur *Lux, Spiritus, Charitas,* de Deo *abſtractè,* ut loquuntur Logici, non concretè, nónne pari analogiâ in re prorsùs incognitâ, “ *conſtituat* quoqùe *durationem* et *ſpatium?*”

200. Ingenti *veræ Theologiæ* detrimento fines *Philoſophiæ Naturalis* (cujus eſt “ fidelitèr Theologiæ ancillari,” ut optimè *Bacon*), temerè tranſgrediuntur Metaphyſici; ubi malè ſani profiteri audent “ *Dei exiſtentiam et attributa demonſtrari à priori*;”—ut infauſtâ diligentiâ, Doctus *Clarke,* ſpeculationibus metaphyſicis ex controverſiâ *Leibnitzianâ* naſcentibus, nimìs imbutus, et iiſdem periculosè indulgens. Deum enim *neceſſariò exiſtere,* certiſſimum eſt: ſed talis exiſtentiæ idea, utcunque per ſe poſitiva, noſtro conceptui eſt planè *negativa*; ſolummodò indicans “ *nullam cauſam* ” ejus dari extrinſecùs: cæterùm *ideæ negativæ ſcientiam poſitivam minimè pariunt,* ipſo teſtante *Locke, Eſſay,* &c. IV. 1. 5.

201. *Exiſtentia Dei* intellectum omnium creaturarum ratione præditarum tranſcendit, imò *Angelorum* et *Archangelorum,* præter “ Filium Charitatis ejus,”—Jesum Christum Dominum nostrum:—qui “ *Illi proximos occupavit honores* ”—*Hor.* ut ex myſtagogo maximo, *Johanne,* diſcimus, *Rev.* xix. 16.

ΒΑΣΙΛΕΥΣ ΒΑΣΙΛΕΩΝ
ΚΑΙ ΚΥΡΙΟΣ ΚΥΡΙΩΝ.

“ Rex Regum et Dominus Dominorum.” (*n*)

Cui

(*n*) Quàm accuratè diſtinguunt Myſtagogi omnium maximi, *Paulus* et *Johannes,* inter Θεον τον Πατερα, “ *Deum Patrem,*” atque ‘Υιον τον Μονογενη, “ *Filium unicè Genuinum,* ſeu *Dilectum,*” *John,* i. 16. *Paulus* enìm Patrem deſignat titulo pleniſſimo maximéque auguſto, ὁ Βασιλευς των Βασιλευοντων, και Κυριος των Κυριοντων, quem haudquaquàm aſſequitur exilitas Linguæ *Latinæ,* articulo *emphatico,* ὁ, carentis; ſed qui *Anglicè* efferri poteſt—“ The King of the reigning [*Kings*], and Lord of the ruling [Lords.]”—*Johannes* verò Filium, titulo planè proximo: Βασιλευς Βασιλεων, και Κυριος Κυριων—“ King of Kings, and Lord of Lords,”—*uno eadémque* Spiritu Divino ambo afflati.

Nec

Cui inditum eſt Dei gratiâ, Ονομα το ὑπερ ϖαν ονομα, "*Nomen ſuprà omne nomen*" (vel auctoritatem) tùm in *Cælis*, tùm in *Terris*, tùm in *Infernis*,—εις δοξαν ΘΕΟΥ ΠΑΤΡΟΣ, "*in laudem* DEI PATRIS;" nunc, ſub *novo fædere*, *Phil.* ii. 9; ſicùt olìm, ſub *primo fædere*, *Exod.* xxiii. 21.—Nam aſſerit ipſe ʽΟ ΛΟΓΟΣ ΤΟΥ ΘΕΟΥ, "*Oraculum Dei*:" *Matt.* xi. 27.

Παντα ΜΟΙ ϖαρεδοθη ὑπο ΤΟΥ ΠΑΤΡΟΣ,
Και ουδεις επιγινωσκει ΤΟΝ ʽΥΙΟΝ, ει μη ʽΟ ΠΑΤΗΡ·
Ουδε ΤΟΝ ΠΑΤΕΡΑ τις επιγινωσκει, ει μη ʽΟ ʽΥΙΟΣ,
ΚΑΙ ᾧ εαν βουληται ʽΟ ʽΥΙΟΣ αποκαλυψαι.

"*Omnia* MIHI *commiſſa ſunt à* PATRE;
Et nemo *intimè cognoſcit* FILIUM, niſi PATER:
Neque PATREM quis *intimè cognoſcit*, niſi FILIUS,
Et cuicunque *voluerit* FILIUS *revelare*."

Nec *Ethnicos* latuit hæc diſtinctio: Audi *Horatium*.

"*Quid priùs dicam ſolitis* PARENTIS
Laudibus, qui res Hominum ac Deorum,
Qui Mare et Terras, variiſque Mundum
Temperat horis?
Unde nil majus generatur IPSO:
Nec viget quicquam ſimile aut ſecundum.
Proximos ILLI *tamen occupavit*
PALLAS *honores*."

Hæc certè ex *Theologiæ primæ* reliquiis extraxit *Horatius*.—"PARENS" enìm eſt ὁ Πατηρ Ανδρωντε Θεωντε, *Orphei*, *Homeri* et *Heſiodi*, licet quid hâc appellatione maximè honorificâ verè ſignificaretur parùm callebant: PALLAS verò, gentium *Dea Bellatrix*, à Παλλω, "*vibro*," [*fulmen*, *telum*, &c.] derivatur; hæc enìm vox *epitheton* tantùm deſignat—Παλλας Αθηνη, ut *Homero* paſsìm. Ejus autèm nomen Αθηνη ab Egyptiacâ Νηϊθ, ſeu SAPIENTIA, retrò legendo derivatur; ut *Not.* (*p*) monuimus: quæ eadem primitùs fuit cum דבר יהוה, *Dabar Iahoh* Teſtamenti Veteris, *Gen.* xv. 1. ſeu Novi—ʽΟ ΛΟΓΟΣ ΤΟΥ ΘΕΟΥ, "*Oraculum Dei*," *Rev.* xix. 13.—ΕΚΕΙΝΟΣ εξηγησατο—"ILLE *expoſuit*."—*John*, i. 18.

Notatu digniſſimum eſt quòd, referente *Horatio*, PALLAS, non *arripuit*, non *arrogavit* ſibi, ſed DEI gratiâ, "*Proximos* ILLI *occupavit* honores,"—*occupavit*, quaſi *jure hæreditario*; ſicùt optimè exponit *Paulus*, de JESU CHRISTO—ʽΟς εν μορφη Θεου ὑπαρχων, ουκ᾽ ἁρπαγμον ἡγησατο το ειναι Ισα-Θεῳ· αλλ᾽ ἑαυτον εκενωσε, &c.—"Qui in *formâ divinâ* [licet] ſubſiſtens, non *inſtar prædæ* duxit *fieri Deo ſimilem*; ſed ſe-ipſum *exhauſit*," &c.—*Phil.* ii. 7.

Fabulantur Mythologi Ethnici, DEAM SAPIENTIAM panopliâ inſtructam εκ Διος κεφαλης, "*è Jovis capite*" emicuiſſe, plenè adultam, non infantem; temerè confundentes, ut videtur, vocem Hebræam ראש *Raoſh*, ambiguè ſignificantem tùm *caput* tùm *principium*, in ſublimi illâ deſcriptione ortûs *Sapientiæ primævæ*, *Prov* viii. 22.

"DOMINUS *genuit* ME, *principium viæ ſuæ*,
Ante opera ſua, primitùs:
Ab æternitate ordinatus ſum, à capite,
A primordiis Terræ."———

Ubi phraſis Hebræa lineæ penultimæ, מראש *Meraŏſh*, quæ ad literam redditur, "*à capite*," ſeu "*ex capite*," æquipollet phraſi lineæ primæ, ראשית דרכו *Raŏſh derechu*—"*principium viæ ſuæ*;"—quam optimè interpretatur *Johannes*, *Rev.* iii. 14.—ʽΗ αρχη της κτισεως Θεου, "*Primordium creationis Dei*," ſeu ϖρωτοτοκος ϖασης κτισεως, '*Primogenitus omnis creationis*," *Coloſſ* i. 15.—De JESU CHRISTO. Et pari ambiguitate, phraſim *Orphei*, *Ζευς κεφαλη*, mutat *Plutarchus* in Ζευς αρχη. *Not.* (*g*).

202. Non niſi *ſtultiſſimi* igitùr autumant ſe *à priori* de Deo Summo philoſophari *Carteſiani*, *Leibnitziani*, *Clarkiani* et *Unitarii*; quorum idcircò—" *dubitans, circumſpectans, hæſitans, multa adverſa revertens, tanquàm in rate in mari immenſo, omnis vehitur oratio*,"—(ut plorat *Cicero* de *Mythologis* antiquis):—" *Relegens errata retrorsùm littora*"—*Virgil.*—" *Inſanientis dum Sapientiæ conſulti errant*;—et *Retrorsùm vela dare, atque iterare curſus coguntur relictos.*" *Hor.*

" *Quantò ſatiùs eſt ire apertâ viâ et rectâ* [Theologiæ Revelatæ], *quàm* tibi ipsi *flexus diſponere; quos cum magnâ moleſtiâ debeas relegere?*" *Seneca.*

203. Graviſſiméque perſtringit *Paulus* hallucinationem inexcuſabilem veterum philoſophantium (*o*): *Rom.* i. 19.

Διοτι το γνωστον ΤΟΥ ΘΕΟΥ *φανερον εστιν εν αυτοις·* Ὁ *γαρ* ΘΕΟΣ *εφανερωσε αυτοις (τα γαρ αορατα* Αυτου, *απο κτισεως* Κοσμου, *τοις ποιημασι νοουμενα καθορᾶται·* Ἥτε *αιδιος* Αυτου ΔΥΝΑΜΙΣ *και* ΘΕΙΟΤΗΣ) *εις το ειναι αυτους αναπολογητους.*

"Quocircà, quicquid *ſcibile* de Deo, manifeſtum eſt inter illos: Deus enìm manifeſtabat illis (nam ab uſque mundo condito, [*attributa*] ejus *inviſibilia*;—nempè æterna ejus potentia et deitas,—*operibus intelligenda*, cernere erat), ut illi forent *inexcuſabiles*."

204. Paritérque *Johannes* *ακαταληψιαν* Mundi docet:

Και ΤΟ ΦΩΣ *εν τη Σκοτια φαινει,*
Και Ἡ Σκοτια *αυτο ου κατελαβεν.*

" Et Lux in *Caligine* lucet;
Sed *Caligo* eam *non comprehendebat.*"

Etsì geſtiebant Veteres, docente *Paulo*: *Act.* xvii. 27.

Ζητειν ΤΟΝ ΚΥΡΙΟΝ, *ει αρα ψηλαφησειαν*
Αυτον, *και ἑυροιεν·*

—" *Quærere* Dominum, ſi fortè *palparent* Illum,
Atque invenirent."

205. Teſtante itèm *Socrate* (Græcorum ſapientiſſimo, ſecundùm *Oraculi Delphici* effatum, quòd, *ſe nihil ſcire præter ſuam inſcitiam de rebus divinis*, profeſſus eſt), in *Phædone*, undè *Paulus* proculdubiò excerpſit:

(*o*) Turpiſſimè hallucinatur vel ipſe *Ariſtoteles*, referente *Cicerone*, *Nat. Deorum*, II. 16. qui, ex *ordine* et *conſtantiâ* motuum cœleſtium, aſtris ipſis *ſenſum* atque *intelligentiam* ſtultiſſimè tribuit!

—" *Ordo* autem aſtrorum, et in *omni æternitate conſtantia*, neque *naturam* ſignificat, ut enim plena rationis; neque *fortunam*, quæ, amica *varietati*, conſtantiam reſpuit: ſequitur ergò, ut ipſa *ſuâ ſponte*, *ſuo ſenſu*, ac *divinitate* moveantur."—

" Cœleſtium ergò *admirabilem ordinem*, *incredibilémque conſtantiam*, ex quâ *conſervatio* et *ſalus* omnium omnis oritnr, qui *vacare mente* putat, is ipſe *mentis expers* habendus eſt." *Nat. Deor.* II. 21.

To

Το γαρ μη διελεσθαι οἱον τ' ειναι, ὁτι αλλο μεν τι εστι το αιτιον τω οντι, αλλο δ' εκεινο ανευ ου το αιτιον ουκ αν ποτ' ειη αιτιον· ὁ δη μοι φαινονται ψηλαφωντες οἱ πολλοι, ὡσπερ εν σκοτει αλλοτριω ομματι προσχρωμενοι, ὡς αιτιον αυτο προσαγορευειν.

"Nam diſtinguendi non capax eſſe, quod unum quidem eſt *ipſa rei alicujus exiſtentis cauſa*, aliud verò, *illa*, *ſine quâ non foret unquam cauſa*: quoad hoc ſanè, mihi videntur pleríque, *palpantes quasì in caligine*; alienis oculis uſi, ſiquidèm *cauſam ipſam*, [immediatam, ultimam falsò] appellant." (*p*).

206. Optiméque et luculentiſſimè quidèm exponit *Paulus*, argumentum illud acutiſſimum omniúmque invictiſſimum quo *Newtonus*, ex "*rerum variatione*," et *tempeſtatum diverſitate*, Numen Supremum colligit: *Act.* xiv. 17.

(*p*) Præclarum quaſi *Socratis* teſtimonium de hallucinatione veterum philoſophorum Dei Summi naturam inveſtigare ſatagentium, affert *Lucianus*, in *Halcyone*, Vol. I. p. 179.

Ω φιλε Χαιρεφων, εοικαμεν ἡμεις των δυνατων τε και αδυνατων ΑΜΒΛΥΩΠΟΙ τινες ειναι κριται παντελως. Δοκιμαζομεν γαρ δη, κατα δυνάμιν ανθρωπινην, [ΤΗΝ ΔΥΝΑΜΙΝ] ΑΓΝΩΣΤΟΝ· ουσαν και ΑΠΙΣΤΟΝ και ΑΟΡΑΤΟΝ Πολλα ουν φαινεται ἡμιν και των ευπορων απορα, και των εφικτων ανεφικτα· συχνα μεν και δι' απειριαν, συχνα δε και δια νηπιοτητα φρενων· τω οντι γαρ νηπιος εοικεν ειναι πας ανθρωπος, και ὁ πανυ γερων, επειτα μικρος πανυ και νεογιλος ὁ του βιου χρονος, προς τον παντα αιωνα. Τι δαν, ὦ 'γαθε, οἱ αγνοουντες τας ΤΩΝ ΘΕΩΝ και ΔΑΙΜΟΝΩΝ ΔΥΝΑΜΕΙΣ, εχοιεν αν ειπειν ποτερον δυνατον η αδυνατον τι των τοιουτων.

"O chare Chærophon, *Nos* quidem *tàm poſſibilium quàm impoſſibilium* videmur eſſe *Judices* HALLUCINANTES prorsùs. Æſtimamus enìm, *ſecundum potentiam humanam*, [Potentiam] NON SCIBILEM, INCREDIBILEM, INVISIBILEM: Multa igitùr ex facilibus difficilia, et ex attingendis non attingenda, nobis apparent. Crebrò quidèm *propter imperitiam*, crebrò autèm *propter infantiam mentis*: Reverà enìm *Infans* videtur eſſe *omnis homo*, etiamſi valdè ſenex; quoniam exiguum prorsùs et puſillum tempus eſt vitæ, cum omni ævo comparatum. Quâ ratione autem, O bone! poſſint, qui neſciunt Deorum Dæmonúmque POTENTIAS, dicere *utrum talium aliquid ſit poſſibile vel impoſſibile?*"

Atque hinc optimè exponenda Inſcriptio Altaris *Athenis*, *Act.* xvii. 23.

ΑΓΝΩΣΤΩ ΘΕΩ.

"Non scibili Deo."

Inſcriptio forſàn derivanda ab Inſcriptione Templi *Nëith* ſeu *Sapientiæ* apud *Sais* in Ægypto Superiore, unde primitùs Colonia *Athenis* deducta fuerat. Et forſàn vox *Athenæ* (Αθηνα), ab Ægyptiâ Νηϊθ vel Νηϊθας, ſi ordine retrogrado legatur, optimè deduci poteſt; ſicut conjecit doctiſſimus *Cudworth*, *Intellect. Syſt.* p. 309, ex *Platone* in *Timæo*, referens:

'Οτι της πολεως [Σαις] Θεος αρχηγος εστιν Αιγυπτιστι μεν τουνομα Νηϊθ, 'Ελληνιστι δε, ὡς ὁ εκεινων λογος, Αθηνα.

"Urbis [*Sais*] *Deus Præſul*, eſt *Nëith*, nomine Ægyptiaco; *Athena* autem, Græco; ſicùt eorum ſermo."

Inſcriptio *Saitica*, a *Plutarcho* ſervatur:

Εγω ειμι ΠΑΝ, το ΓΕΓΟΝΟΣ, και ΟΝ, και ΕΣΟΜΕΝΟΝ.
Και τον εμον πεπλον ουδεις πω Θνητος απεκαλυψεν.

"*Ego ſum* Omne, *quod* FUIT, *quod* EST, et *quod* ERIT:
Atque meum peplum nullus adhuc Mortalis revelavit."

Atqui nemo quidem Mortalium revelare potuit, donec Ipſe demiſſus eſt e ſinu Patris, ὁ μονογενης, "*unicè dilectus* Filius, qui naturam cultúmque Dei cæcutientibus mortalibus exponeret." *John* i. 18.

Καιτοιγε ȣκ αμαρτυρον ‘Εαυτον αφηκεν (‘Ο ΘΕΟΣ) αγαθοποιων, ȣρανοθεν ἡμιν ὑετȣς διδȣς και καιρȣς καρποφυρȣς, εμπιπλων τρυφης και ευφροσυνης τας καρδιας ἡμων.

“ Veruntamèn, ſe ipſum *haud inteſtatum* relinquebat (Deus) utique *benefaciendo*, cœlitùs *imbres* nobis ac *tempeſtates fructuoſas præbendo*, corda noſtra *cibo* atque *lætitiâ* implendo.”

Haud raró enim omnia fiunt contraria: puniuntur immemorum atque ingratorum corda, ultione divinâ, regionum ſiccitate, tempeſtatibus infructuoſis, fame et pavore palpitantium:

> “ *Terra enim ferax in ſalſuginem convertitur,*
> *Propter iniquitatem incolentium ejus.*”

Atqui hoc argumentum ex *cauſis finalibus* petitum, omnium conceptibus accommodatiſſimum, vix, et ne vix quidem, attigerunt Sophiſtæ veteres; dum *Legibus Naturæ generalibus*, ſecundùm quas ordinantur anni cardines tempeſtatûmque viciſſitudines revolutionibus periodíſque certis et æquabilibus, *mechanicè* quaſi agentibus, tantùm incumbentes, CAUSÆ PRIMARIÆ obliti, in praviſſimum *Atheiſmum* delapſi ſunt.—Vide *Ariſtotelem*, aſtris (propter ordinem et conſtantiam motuum quibus in cœlis circumferri obſervantur,) ſenſum et intelligentiam aſſerentem.—*Nat. Deor.* II. 16.

“ *Extat igitur*, ut præclarè loquitur *Coteſius*, “ *eximium* NEWTONI OPUS *adversùs* ATHEORUM *impetus munitiſſimum præſidium:* neque enim alicunde feliciùs, [ORACULIS DIVINIS *exceptis*], *quàm ex hâc pharetrâ, contrà impiam catervam tela depromſeris.*”—*Præfat.* ad ſecundam editionem Principiorum Newtoni, anno 1713 à Coteſio factam.

207. Coronidis loco ſubjungere libet ſententiam auream cedróque dignam, quam protulit magnus ille BACON, *Philoſophiæ Experimentalis Pater*, et NEWTONI *præceptor:*

“ A *ſuperficial taſte* of PHILOSOPHY may perchance incline the mind to ATHEISM; but a *full draught* thereof brings it back again to RELIGION.

“ In the entrance of Philoſophy, when the *ſecond cauſes*, moſt obvious to the ſenſes, offer to the mind, we are apt to cleave unto them, and dwell too much upon them; ſo as to *forget* what is ſuperior in Nature: but when we paſs further, and behold the *dependency* and *confederacy of cauſes*, and the *works* of PROVIDENCE; then, according to the Poets, we eaſily perceive, that *the higheſt link of* NATURE*'s chain muſt be tied to the foot of* JUPITER*'s chair—that* PHILOSOPHY, *like* JACOB*'s viſion, diſcovers to us a* LADDER, *whoſe top reaches up to the footſtool of the throne of* GOD.”

HINC OMNE PRINCIPIUM, HUC REFER EXITUM.

Typis J. DAVIS, in vico dicto Chancery-Lane.

BRITISH MUSEUM 29 FEB 68

35